

NATIONAL UNIVERSITY OF SINGAPORE

Department of Mathematics

2005/2006 Semester I

MA2108 Advanced Calculus II

Tutorial 9

1. Consider the function

$$F(x) = \sum_{k=1}^{\infty} \frac{(-1)^k x^k}{1+x^{2k}}, \quad x \in \left(0, \frac{2}{3}\right).$$

Show that F is continuous on the interval $\left(0, \frac{2}{3}\right)$. [Hint: You may need the Weierstrass M-test.]

2. Let $\sum_{k=1}^{\infty} a_k$ be an absolutely convergent series.

i) Show that $\sum_{k=1}^{\infty} (-1)^k a_k \sin kx$ converges uniformly on $(-\infty, +\infty)$.

ii) Hence evaluate $\int_0^{2\pi} \sum_{k=1}^{\infty} 2(-1)^k a_k \sin kx dx$. Justify your answer.

3. Show that the function $f(x) = \sum_{n=1}^{\infty} \frac{\cos^n x}{n^3}$ is differentiable on $(-\infty, +\infty)$.

4. Find the radius of convergence of each of the following power series:

i) $\sum_{n=1}^{\infty} \left(1 + \frac{3}{n}\right)^{n^2} x^n$.

ii) $\sum_{n=1}^{\infty} \frac{(-3)^n}{n!} (x-1)^n$.

iii) $\frac{x}{5} + \left(\frac{x}{6}\right)^2 + \left(\frac{x}{5}\right)^3 + \left(\frac{x}{6}\right)^4 + \left(\frac{x}{5}\right)^5 + \left(\frac{x}{6}\right)^6 + \dots$.

iv) $\sum_{n=1}^{\infty} \frac{(3x-2)^n}{n^2}$.