Braid Group Action and the Homotopy Groups
(Jie Wu)

The Artin braid group B_n can be combinatorially defined by

generators: $\sigma_1, \sigma_2, \ldots, \sigma_{n-1}$.

relation: $\sigma_i\sigma_j = \sigma_j\sigma_i$ for $|i-j| \geq 2$ and $\sigma_i\sigma_{i+1}\sigma_i = \sigma_{i+1}\sigma_i\sigma_{i+1}$ for each i.

The Artin representation of B_n is as follows. Let $F_n = F(x_1, x_2, \ldots, x_n)$ be the free group of rank n generated by letters $x_1, x_2, \ldots, x_n$. Then B_n is the group of automorphisms of F_n generated by the automorphisms σ_i, $1 \leq i \leq n-1$, given by $\sigma_i(x_j) = x_j$ for $j \neq i, i+1$, $\sigma_i(x_i) = x_{i+1}$ and $\sigma_i(x_{i+1}) = x_{i+1}^{-1}x_ix_{i+1}$.

Let $G(n)$ be the group defined in Problem 1 and let $q_n \colon F_n \to G(n)$ be the quotient homomorphism. Then the Artin representation of B_n on F_n induces a representation of B_n on $G(n)$, that is, for any $\beta \in B_n$ there is a (unique) automorphism $\bar{\beta}$ of $G(n)$ such that $\bar{\beta} \circ q_n = q_n \circ \beta \colon F_n \to G(n)$.

Problem. 1. Determine the set of fixed points of the B_n action on $G(n)$.
2. Determine the set of fixed points of the pure Artin braid group P_n action on $G(n)$.

Any answer of this problem is extremely interesting in homotopy theory because

1) The set of fixed points of B_n action on $G(n)$, as a group, is isomorphic to the subgroup of $\pi_n(S^2)$ consisting of elements of order 2.
2) The set of fixed points of P_n action on $G(n)$, as a group, is isomorphic to $\pi_n(S^2)$.