Supplement to Section 3.10

Example. Evaluate $\sqrt{4.1}$ with error less than 0.001.

Solution.

$$\sqrt{4.1} = \left(4+\frac{1}{10}\right)^{\frac{1}{2}} = 2\left(1+\frac{1}{40}\right)^{\frac{1}{2}} = 2\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(\frac{1}{40}\right)^{k}.$$

$$= 2 + 2\sum_{k=1}^{\infty}\binom{\frac{1}{2}}{k}\left(\frac{1}{40}\right)^{k}.$$

Now

$$\binom{\frac{1}{2}}{k} = \frac{\frac{1}{2}\cdot\left(\frac{1}{2}-1\right)\cdots\left(\frac{1}{2}-k+1\right)}{k!} = (-1)^{k-1}\frac{(2k-3)!!}{k!2^{k}}$$

for $k \geq 2$. The series

$$\sum_{k=1}^{\infty}\binom{\frac{1}{2}}{k}\left(\frac{1}{40}\right)^{k}$$

is an alternating series satisfying the conditions for the alternating series estimation. (check this !!) From

$$2\cdot\left|\binom{\frac{1}{2}}{k+1}\left(\frac{1}{40}\right)^{k+1}\right| = \frac{1}{6400} < 0.001,$$

we have $k \geq 1$ and so

$$\sqrt{4.1} \approx 2 + 2\binom{\frac{1}{2}}{1}\frac{1}{40} = 2.025$$

with error less than 0.001, by the alternating series estimation. □