1. By computing derivatives, find the Taylor series of

i) $f(x) = e^{2x}$ at $x = 3$.

ii) $f(x) = \cos x$ at $x = \dfrac{\pi}{3}$.

2. Find the Taylor series of $\ln(1 + 2x^2)$ at $x_0 = 0$.

3. Using the Taylor Formula, show that $\cos x = \displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$.

4. Use series to estimate the integral's value

$$\int_0^{0.2} \sin x^2 dx.$$

with an error of magnitude less than 10^{-8}.

5. Use series to evaluate the limits

i) $\displaystyle\lim_{y\to 0} \frac{\arctan y - \sin y}{y^3 \cos y}$.

ii) $\displaystyle\lim_{x\to\infty} x^2(e^{-1/x^2} - 1)$.

Review question of chapter 3.

6. Let $\displaystyle\sum_{n=1}^{\infty} f_n(x)$ be a series of functions on an interval I and let $\{g_n(x)\}$ be a sequence of functions on I. Suppose that

1) $\displaystyle\sum_{n=1}^{\infty} |f_n(x)|$ converges uniformly on I and

2) there exists a positive number M such that $|g_n(x)| \le M$ for all $x \in I$ and all $n \ge 1$.

Show that the series of functions $\displaystyle\sum_{n=1}^{\infty} f_n(x)g_n(x)$ converges uniformly on I.