

This is the last tutorial. The questions with \* will be gone through during lectures.

1. ‘Classify’ the following differential equations, i.e., determine whether the following equations are homogeneous/non-homogeneous, linear/non-linear, 1st order/2nd order/..., ordinary differential equations:

a)  $t \frac{dy}{dt} + y \sin t = 0$ ;

b)  $(1 + t^2) \frac{dy}{dt} + 2ty + 3 = 0$ ;

c)  $\frac{dy}{dt} + \frac{d^3y}{dt^3} \frac{dy}{dt} = 3t$ ;

d)  $4 \frac{d^2y}{dt^2} + 5 \frac{dy}{dt} + 6y = e^t + \sin 2t$ ;

e)  $u \frac{d^4u}{dt^4} + \left( \frac{du}{dt} \right)^2 = 0$ .

2. Find the general solution of the following differential equations:

a)  $\frac{dy}{dt} + y \sin t = 0$ ;

b)  $(1 + t^2) \frac{dy}{dt} + ty = (1 + t^2)^{3/2}$

3. Solve the following initial/boundary value problems, and express your answers in the form “ $y=y(t)$ ”.

a)  $\frac{dy}{dt} + 4ty = 2te^{-2t^2}$ ,  $y(1) = 5$ .

b)\*  $(1 + t^2) \frac{dy}{dt} - 2ty^4 = 0$ ,  $y(0) = \frac{1}{2}$ .

c)\*  $\sqrt{1 + t^3} \frac{dy}{dt} - 6t^2 e^{-y} = 0$ ,  $y(0) = 0$ .

4. Find the general solution of the following differential equations

a)  $\frac{dy}{dt} = \frac{y}{t} + \frac{y^2}{t^2}$ .

b)\*  $\frac{dy}{dt} = \frac{y}{t} - \tan \frac{y}{t}$ .

5. Find the the general solution of the following differential equations:

a)  $\frac{d^2y}{dt^2} - 7 \frac{dy}{dt} + 10y = 0$ ;

b)  $\frac{d^2y}{dt^2} + 6 \frac{dy}{dt} + 9y = 0$ ;

c)  $\frac{d^2y}{dt^2} + 2 \frac{dy}{dt} + 3y = 0$ .

6. Solve the following initial value problem:

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y = 0; \quad y(0) = 1, \quad y'(0) = 0.$$

7. Use the method of undetermined coefficients to find a particular solution of the following differential equations:

a)  $y'' + 3y = 6t^3 - 6.$

b)\*  $y'' + 3y = \cos(\sqrt{3}t).$

8\*. Find the general solution of the following differential equations:

a)  $y'' - 3y' + 2y = 3e^{2t} + e^{3t}.$

b)  $y'' + 4y = \cos 2t + \sin t.$

c)  $y'' - 2y' + y = \frac{e^t}{t}.$

9\*. Solve the following initial value problem:

$$y'' + 4y' + 3y = 2e^{-3t}; \quad y(0) = 0, \quad y'(0) = 3.$$

10\*. A tank containing  $S_0$  lb of salt dissolved in 200 gallons of water. Starting at time  $t = 0$ , water containing  $\frac{1}{2}$  lb of salt per gallon enters the tank at rate of 4 gal/min, and well stirred solution leaves the tank at the same rate. Find the concentration of salt in the tank at any time  $t > 0$ .

Some suggested answers:

1 a) homogeneous linear 1st order ODE

1 b) non-homogeneous linear 1st ODE

1 c) non-homogeneous non-linear 3rd order ODE

1 d) non-homogeneous linear 2nd order ODE

1 e) homogeneous non-linear 4-th order ODE.

2 a)  $y = Ce^{\cos t}$

2 b)  $y = \frac{1}{\sqrt{1+t^2}} \left( t + \frac{t^3}{3} + C \right)$

3 a)  $y = e^{-2t^2} (t^2 + 5e^2 - 1)$

3 b)  $y = \frac{1}{\sqrt[3]{8 - 3\ln(1+t^2)}}$

3 c)  $y = \ln(4\sqrt{1+t^3} - 3)$

4 a)  $y = -\frac{t}{\ln(|t|) + C}$

4 b)  $y = t \arcsin \left( \frac{C}{t} \right)$

5 a)  $y(t) = Ae^{2t} + Be^{5t}$

5 b)  $y(t) = (At + B)e^{-3t}$

5 c)  $y(t) = Ae^{-t} \cos \sqrt{2}t + Be^{-t} \sin \sqrt{2}t$

6.  $y = e^{-t} + te^{-t}$

7 a)  $y_p(t) = 2t^3 - 4t - 2$

7 b)  $y_p = \frac{1}{2\sqrt{3}}t \sin(\sqrt{3}t)$

8 a)  $y(t) = Ae^t + Be^{2t} + 3te^{2t} + \frac{1}{2}e^{3t}$

8 b)  $y(t) = A \cos 2t + B \sin 2t + \frac{t}{4} \sin 2t + \frac{1}{3} \sin t$

8 c)  $y(t) = Ae^t + Bte^t + te^t \ln |t|$

9.  $y(t) = 2e^{-t} - 2e^{-3t} - te^{-3t}$

10. the concentration  $y(t)$  of salt in the tank is given by

$$y(t) = \frac{S_0}{200}e^{-0.02t} + \frac{1}{2}(1 - e^{-0.02t}).$$