

- Let  $\{a_n\}$  and  $\{b_n\}$  be Cauchy sequences. Show that  $\{a_n + b_n\}$  and  $\{a_n b_n\}$  are also Cauchy sequences.
- For each of the following series, calculate the  $n$ -th partial sum  $S_n$ , and determine whether the series is convergent or divergent

i)  $\sum_{n=1}^{\infty} \ln \frac{n+2}{n+3}$ .

ii)  $\sum_{n=1}^{\infty} \frac{1}{n(n+2)}$ .

- Determine the convergence or divergence of each of the following series. Justify your answers.

(a).  $\sum_{n=1}^{\infty} \frac{n^2 - 1}{2n^2 + n}$ .

(b).  $\sum_{n=1}^{\infty} \sin \frac{n\pi}{2}$ .

(c).  $\sum_{n=1}^{\infty} \frac{n^2 + 1 + \ln n}{n + n^3 + 4}$ .

(d).  $\sum_{n=1}^{\infty} \frac{3 + \sin n}{n^2}$ .

(e).  $\sum_{n=1}^{\infty} \frac{2^n + 3}{3^{n+1} - n}$ .

(f).  $\sum_{n=1}^{\infty} \frac{2}{n^{1+\frac{1}{n}}}$ .

(g).  $\sum_{n=1}^{\infty} \frac{4 + (-1)^n}{2n}$ .

(h).  $\sum_{n=2}^{\infty} \frac{1}{n(1 + \ln n)^p}$  with  $p \leq 0$ .

(i).  $\sum_{n=1}^{\infty} \frac{n}{n^2 + 1}$ .

**Some suggested answers:**

2. i)  $S_n = \ln 3 - \ln(n+3)$  and  $\sum_{n=1}^{\infty} \ln \frac{n+2}{n+3}$  is divergent.

2. ii)  $S_n = \frac{1}{2} \left[ \frac{3}{2} - \frac{2n+3}{(n+1)(n+2)} \right]$  and  $\sum_{n=1}^{\infty} \frac{1}{n(n+2)} = \lim S_n = \frac{3}{4}$ .

3. a) divergent by the divergence test.

3. b) divergent by the divergence test.
3. c) divergent by the limit comparison test (comparing with  $\sum_{n=1}^{\infty} \frac{1}{n}$ ).
3. d) convergent by the comparison test (comparing with  $\sum_{n=1}^{\infty} \frac{4}{n^2}$ ).
3. e) convergent by the limit comparison test (comparing with  $\sum_{n=1}^{\infty} \left(\frac{2}{3}\right)^n$ ).
3. f) divergent by the limit comparison test (comparing with  $\sum_{n=1}^{\infty} \frac{1}{n}$ ).
3. g) divergent by the comparison test (comparing with  $\sum_{n=1}^{\infty} \frac{3}{2n}$ ).
3. h) divergent by the comparison test (comparing with  $\sum_{n=1}^{\infty} \frac{1}{n}$ ).
3. i) divergent by the limit comparison test (comparing with  $\sum_{n=1}^{\infty} \frac{1}{n}$ ).