1. (a). Let $\sum_{n=1}^{\infty} a_n$ be a convergent series with each $a_n \geq 0$. Show that the series $\sum_{n=1}^{\infty} a_n^2$ is also convergent.
   [Hint: First show that $|a_n| < 1$ for n sufficiently large, and then use the comparison test.]

   (b). Give an example of a convergent series $\sum_{n=1}^{\infty} a_n$ such that $\sum_{n=1}^{\infty} \sqrt{a_n}$ diverges.

2. Use the integral test to determine the convergence or divergence of the series $\sum_{n=1}^{\infty} \frac{1}{n(1+\ln n)}$.

3. Use the ratio test to determine the convergence or divergence of each of the following series.

   (a). $\sum_{n=1}^{\infty} \frac{(3n)!}{6^n n!(2n)!}$

   (b). $\sum_{n=1}^{\infty} a_n$, where $a_1 = 1$, $a_n = 2\left(1 - \frac{1}{n}\right)^n a_{n-1}$, $n = 2, 3, \cdots$.

4. Use the (simplified) root test to determine the convergence or divergence of each of the following series.

   (a). $\sum_{n=1}^{\infty} \frac{5n^2 \cdot 3^n}{4^{n+4}}$.

   (b). $\sum_{n=1}^{\infty} \frac{3^{2n}}{5^n}\left(1 - \frac{1}{2n}\right)^{n^2}$.

   (c). $\frac{1}{4} + \frac{1}{5^2} + \frac{1}{4^3} + \frac{1}{5^4} + \frac{1}{4^5} + \frac{1}{5^6} + \frac{1}{4^7} + \frac{1}{5^8} + \cdots$.

Some suggested answers:

2. divergent.
3(a). divergent.
3(b). convergent.
4(a). convergent.
4(b). divergent.
4(c). convergent.