

1. Determine the convergence or divergence of each of the following series. Justify your answers.

$$(a). \sum_{n=1}^{\infty} (\sqrt{2n+2} - \sqrt{n}).$$

$$(b). \sum_{n=1}^{\infty} \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{n!} \cdot \frac{2^n}{5^n}.$$

$$(c). \sum_{n=1}^{\infty} \frac{\ln n}{n^{1.2}}.$$

$$(d). \sum_{n=1}^{\infty} \left(\frac{n}{n+2} \right)^{n^2}.$$

$$(e). \sum_{n=2}^{\infty} \frac{1}{(\ln n)^3}.$$

$$(f). \sum_{n=1}^{\infty} \left(\frac{4}{9} + \frac{n^3}{3^n} \right)^{\frac{n}{2}}.$$

2. Consider the series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{\ln n}{\sqrt{n}}$.

- i) Use the alternating series test to show that the series is convergent.
- ii) Using part i) or otherwise, show that the series is conditionally convergent.

3. Show that the series $\sum_{n=1}^{\infty} (-1)^n \frac{\cos n}{2^n}$ is absolutely convergent.

4. For each of the following series, determine whether the series is absolutely convergent, conditionally convergent or divergent. Justify your answers.

$$(a). \sum_{n=1}^{\infty} (-1)^n \frac{3}{2n+1}.$$

$$(b). \sum_{n=1}^{\infty} (-1)^n \frac{n}{4n+3}.$$

$$(c). \sum_{n=1}^{\infty} (-1)^n \left(\frac{1+2n}{3+4n} \right)^n.$$

$$(d). \sum_{n=2}^{\infty} (-1)^{n+1} \frac{1}{n \ln n}.$$

Some suggested answers:

- 1(a). divergent.
- 1(b). convergent.
- 1(c). convergent.
- 1(d). convergent.
- 1(e). divergent.
- 1(f). convergent.
- 4(a). conditionally convergent.
- 4(b). divergent.
- 4(c). absolutely convergent.
- 4(d). conditionally convergent.