1. Estimate the infinite sum $\sum_{n=1}^{\infty} \frac{1}{n^5}$ such that the error is within 0.001.
2. Estimate the infinite sum $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n^5}$ such that the error is within 0.001.
3. For each of the following sequence of functions, determine whether is converges pointwise to a function, and find the limiting function if it exists. Justify your answers.
   (a). $\left\{ \left(1 + \frac{x}{n}\right)^{nx} \right\}$, $x \in (-\infty, +\infty)$.
   (b). $\{x^{n+1}\}$, $x \in [-1, 1]$.
   (c). $\left\{ \frac{x^{2n}}{1 + x^{2n}} \right\}$, $x \in [0, 1]$.
4. Determine whether the following sequences of functions converge uniformly on the indicated intervals. Justify your answers.
   (a). $F_n(x) = \frac{n^2}{n^2 + x^2}$, $x \in [0, 1]$.
   (b). $F_n(x) = x^n(1 - x)$, $x \in [0, 1]$.
   (c). $f_n(x) = \frac{n \ln x}{x^n}$, $x \in [1, \infty)$.
   (d). $f_n(x) = \frac{n \ln x \cos nx}{x^n}$, $x \in [4, \infty)$.
   (e). $F_n(x) = \frac{n^2}{n^2 + x^2}$, $x \in [0, +\infty)$.
5. Let $\{F_n\}$ be a sequence of functions on an interval I. It is given that $\{F_n\}$ converges uniformly on some function F on I. Suppose also that for each $n \in \mathbb{Z}^+$, there exists a real number $M_n > 0$ such that
   $$|F_n(x)| \leq M_n \quad \text{for all } x \in I.$$
   Show that there exists a real number M such that $|F(x)| \leq M$ for all $x \in I$.