1. Evaluate the limits. Justify your answers.

i) $\lim_{n\to\infty} \int_0^1 \frac{n+e^x}{n+x^2} dx.$

ii) $\lim_{n\to\infty} \int_1^2 \left(\frac{x^2+1}{8}\right)^n \sin nx dx.$

2. Prove that each of the following series of functions converges uniformly on the indicated interval.

i) $\sum_{n=1}^{\infty} \frac{\cos nx}{n^2+x^2}$, $x \in (-\infty, +\infty)$.

ii) $\sum_{n=1}^{\infty} \frac{1}{1+n^3x^2}$, $x \in [2, \infty)$.

iii) $\sum_{n=1}^{\infty} \frac{xe^{-nx}}{n^2}$, $x \in (0, \infty)$.

3. Let $\sum_{n=1}^{\infty} f_n(x)$ and $\sum_{n=1}^{\infty} g_n(x)$ be series of functions on an interval I with

$$|f_n(x)| \le g_n(x)$$

for all $x \in I$ and $n \ge 1$. Suppose that the series of functions $\sum_{n=1}^{\infty} g_n(x)$ converges uniformly. Show that the series of functions $\sum_{n=1}^{\infty} f_n(x)$ converges uniformly.