1. Consider the function

$$F(x) = \sum_{k=1}^{\infty} \frac{(-1)^k x^k}{1+x^{2k}}, \quad x \in \left(0, \frac{2}{3}\right).$$

Show that F is continuous on the interval $\left(0, \frac{2}{3}\right)$.

2. Evaluate $\sum_{n=0}^{\infty} \int_0^{\frac{1}{2}} \frac{x^n(1-x^2)}{\sqrt{1+x}} dx$ in simplest form. Justify your answer.

3. Let $\sum_{k=1}^{\infty} a_k$ be an absolutely convergent series.

i) Show that $\sum_{k=1}^{\infty} a_k \sin kx$ converges uniformly on $(-\infty, +\infty)$.

ii) Hence evaluate $\int_0^{2\pi} \sum_{k=1}^{\infty} a_k \sin kx dx$. Justify your answer.

4. By using the formulate $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$, estimate the integral $\int_0^1 e^{-x^3} dx$ such that the error is within 0.001.

5. Show that the function $f(x) = \sum_{n=1}^{\infty} \frac{\cos^n x}{n^3}$ is differentiable on $(-\infty, +\infty)$.

6. Find the radius of convergence of each of the following power series:

i) $\sum_{n=1}^{\infty} \left(1 + \frac{3}{n}\right)^{n^2} x^n$.

ii) $\sum_{n=1}^{\infty} \frac{(-3)^n}{n!}(x-1)^n$.

iii) $\frac{x}{5} + \left(\frac{x}{6}\right)^2 + \left(\frac{x}{5}\right)^3 + \left(\frac{x}{6}\right)^4 + \left(\frac{x}{5}\right)^5 + \left(\frac{x}{6}\right)^6 + \cdots$.

iv) $\sum_{n=1}^{\infty} \frac{(3x-2)^n}{n^2}$.