

1. Find the interval of convergence of each of the following power series:

i) $\sum_{n=1}^{\infty} \frac{(-2x)^n}{n^{\frac{3}{2}}}$.

ii) $\sum_{n=1}^{\infty} \frac{3^n(x-2)^n}{n+1}$.

iii) $\sum_{n=1}^{\infty} \frac{(1-3x)^n}{n}$.

2. By integrating from $t = 0$ to $t = x$ the power series $\frac{1}{1+t} = \sum_{n=0}^{\infty} (-1)^n t^n$, $|t| < 1$, show that

$$\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^n}{n}$$

for all $|x| < 1$.

3. Show that $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$ ($|x| < \infty$)

4. Use series to estimate the integral' value

$$\int_0^{0.1} \frac{1}{\sqrt{1+x^4}} dx$$

with an error of magnitude less than 10^{-8} .

5. Let $f(x) = \sqrt[5]{1+x^3}$. Find $f^{(30)}(0)$.

6. Estimate $\sqrt[3]{9}$ with an error of magnitude less than 10^{-4} .