

## Take-home Exam 6

In order to get the full marks for the project of take-home exams, you need to have 50 or more points. Well there are 10 points in this take-home exam.

**Question 1.** [4 points, 1 for each part] For each of the following, determine all values of  $x$  for which the given series converges.

- (a) 
$$\sum_{k=1}^{\infty} \frac{(1-2x)^k}{k}.$$
- (b) 
$$\sum_{k=1}^{\infty} \frac{3^k}{k^3} (2x+1)^k.$$
- (c) 
$$\sum_{k=1}^{\infty} \frac{3^k x^k}{2^k (1-x)^k}, \quad x \neq 1.$$
- (Hint: Let  $t = \frac{3x}{2(1-x)}$ .)
- (d) 
$$\sum_{k=1}^{\infty} \frac{1}{kx^k}, \quad x \neq 0.$$
- (Hint: Let  $t = \frac{1}{x}$ .)

**Question 2.** [4 points, 1 for each part] Using any applicable method, find the Taylor series of each of the following functions at the indicated point, and specify the interval on which the series converges to the function.

- (a)  $f(x) = \cos x^2, \quad x_0 = 0.$
- (b)  $f(x) = \ln \left( \frac{1+x}{1-x} \right), \quad x_0 = 0.$
- (c)  $f(x) = \sqrt{x}, \quad x_0 = 1.$
- (d)  $f(x) = \frac{x^2}{1-x^2}, \quad x_0 = 0.$

**Question 3.** [2 points, 1 for each part]

- (a) Use series to estimate the integral's value

$$\int_0^{0.1} \arctan x^2 dx$$

with an error of magnitude less than  $10^{-8}$ .

- (b) Let  $f(x) = x^3 \sin x^9$ . Find  $f^{(30)}(0)$ .