

1. Denote the set of rational numbers by \mathbb{Q} . Consider the set

$$S = \{x \in \mathbb{Q} \mid 0 \leq x < 1\}.$$

Find $\sup S$ and $\inf S$. Justify your answers.

2. Let A and B be two non-empty bounded set of real numbers such that $A \subseteq B$. Show that $\inf A \geq \inf B$.
3. Let A and B be two non-empty bounded set of real numbers
- Show that $\sup A \cup B = \max\{\sup A, \sup B\}$.
 - Is it true that $\sup A \cap B = \min\{\sup A, \sup B\}$? Justify your answer.
4. Consider the sequence $\{a_n\}$ defined recursively by

$$a_1 = 2, \quad a_n = \sqrt{6 + a_{n-1}}, \quad n = 2, 3, 4, \dots$$

- Show that $2 \leq a_n \leq 3$ for all n .
 - Show that $\{a_n\}$ is monotone increasing.
 - Using parts i) and ii), show that $\{a_n\}$ converges, and find its limit.
5. Consider the sequence $\{x_n\}$ defined recursively by

$$x_1 = \frac{3}{4}, \quad x_{n+1} = 2x_n - x_n^2, \quad n = 1, 2, 3, \dots$$

Show that $\{x_n\}$ converges, and find its limit. (Hint: Show that $0 \leq x_n \leq 1$ for all n and $\{x_n\}$ is monotone increasing.)

6. Find the $\overline{\lim}_{n \rightarrow \infty}$ and $\underline{\lim}_{n \rightarrow \infty}$ of the following sequences.

(a). $\left\{ 4 + \cos \frac{n\pi}{2} \right\}$.

(b). $\left\{ \frac{1 + (-1)^n}{n} \right\}$.