

1. Determine whether the following sequences of functions converge uniformly on the indicated intervals. Justify your answers.

(a). $F_n(x) = \frac{n^2}{n^2 + x^2}, x \in [0, 1]$.

(b). $F_n(x) = x^n(1 - x), x \in [0, 1]$.

(c). $f_n(x) = \frac{n \ln x}{x^n}, x \in [1, \infty)$.

(d). $f_n(x) = \frac{n \ln x \cos nx}{n^2 x^n}, x \in [4, \infty)$.

(e). $F_n(x) = \frac{n^2}{n^2 + x^2}, x \in [0, +\infty)$.

2. Prove that each of the following series of functions converges uniformly on the indicated interval.

i) $\sum_{n=1}^{\infty} \frac{\cos nx}{n^2 + x^2}, x \in (-\infty, +\infty)$.

ii) $\sum_{n=1}^{\infty} \frac{1}{1 + n^3 x^2}, x \in [2, \infty)$.

iii) $\sum_{n=1}^{\infty} \frac{x e^{-nx}}{n^2}, x \in (0, \infty)$.

3. Let $\sum_{n=1}^{\infty} f_n(x)$ and $\sum_{n=1}^{\infty} g_n(x)$ be series of functions on an interval I with

$$|f_n(x)| \leq g_n(x)$$

for all $x \in I$ and $n \geq 1$. Suppose that the series of functions $\sum_{n=1}^{\infty} g_n(x)$

converges uniformly. Show that the series of functions $\sum_{n=1}^{\infty} f_n(x)$ converges uniformly.

4. Does the series of functions $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n^x}$ converge uniformly on the interval $(0, +\infty)$? Justify your answer.