1. Show that the function $f(x) = \sum_{n=1}^{\infty} \frac{\cos^n x}{n^3}$ is differentiable on $(-\infty, +\infty)$.
2. (a). Prove that the function
$$f(x) = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots \qquad (|x| < \infty)$$
is a solution of the differential equation
$$\frac{dy}{dx} = y.$$
(b). By solving the above differential equation, show that
$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots \qquad (|x| < \infty).$$
3. Find the radius of convergence of each of the following power series:
i) $\sum_{n=1}^{\infty} \left(1 + \frac{3}{n}\right)^{n^2} x^n$.
ii) $\sum_{n=1}^{\infty} \frac{(-3)^n}{n!}(x-1)^n$.
iii) $\frac{x}{5} + \left(\frac{x}{6}\right)^2 + \left(\frac{x}{5}\right)^3 + \left(\frac{x}{6}\right)^4 + \left(\frac{x}{5}\right)^5 + \left(\frac{x}{6}\right)^6 + \cdots$.
iv) $\sum_{n=1}^{\infty} \frac{(3x-2)^n}{n^2}$.
4. Estimate $\sqrt[3]{9}$ with an error of magnitude less than 10^{-4}. (Hint: $\sqrt[3]{9} = (8+1)^{\frac{1}{3}} = 2\left(1 + \frac{1}{8}\right)^{\frac{1}{3}}$.)