INTRODUCTION TO ALGEBRAIC TOPOLOGY
TUTORIAL 3

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Problem 1. Let X and Y be pointed spaces. Show that $\Omega^n(X\times Y)\cong\Omega^n(X)\times\Omega^n(Y)$ and $\Omega^{n+m}(X)\cong\Omega^m(\Omega^n(X))$.

Problem 2. Show that the real projective space $\mathbb{R}\mathrm{P}^n$ is an n-manifold. (Hint: $\mathbb{R}\mathrm{P}^n\cong S^n/(\mathbb{Z}/2)$.)

Problem 3. Show that $\mathbb{C}\mathrm{P}^n/\mathbb{C}\mathrm{P}^{n-1}\cong S^{2n}$. In particular, $\mathbb{C}\mathrm{P}^1\cong S^2$.

Problem 4. Show that and the complex projective space $\mathbb{C}P^n$ is a $2n$-manifold.

Problem 5. Let $S^1\subseteq\mathbb{C}$ act on $S^{2n+1}\subseteq\mathbb{C}^{n+1}$ by sclar multiplication, that is

$$\alpha\cdot(z_1,\ldots,z_{n+1})=(\alpha z_1,\ldots,\alpha z_{n+1}).$$

Show that S^{2n+1}/S^1 is homeomorphic to $\mathbb{C}\mathrm{P}^n$. In particular, one gets the famous Hopf fibration that $S^2\cong\mathbb{C}\mathrm{P}^1$ is the quotient of S^3 modulo the S^1-action.

Problem 6. Prove Theorem 3.1.7.