INTRODUCTION TO ALGEBRAIC TOPOLOGY
TUTORIAL 4

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Problem 1. Let $f\colon X \to Y$ be a map. If X is path-connected, then the image $f(X)$ is path-connected

Problem 2. Let X be a non-empty space and let x_0 be any point in X which is regarded as the base point. Then

$$\pi_0(X) \cong (X/\simeq).$$

Problem 3. Let X and Y be topological spaces. Then X and Y are path-connected if and only if $X \times Y$ is path-connected.

Problem 4. The *comb space* Y is defined by

$$Y = \{(x,y) \in \mathbb{R}^2 | 0 \leq y \leq 1, x = 0, 1/n \text{ or } y = 0, 0 \leq x \leq 1\}.$$

Show that the identity map of Y is homotopic to the constant map to $(0,1) \in Y$.

Problem 5. Show that A is a weak deformation retract of X if and only if A is a weak retract of X and X is deformable into A.

Problem 6. Show that S^n is a strong deformation retract of $\mathbb{R}^{n+1} \setminus \{0\}$.

Problem 7. Suppose that X is deformable into a retract A. Show that A is a deformation retraction of X.

Problem 8. Show that $S^n \subseteq D^{n+1}$ is a cofibration.