

# INTRODUCTION TO ALGEBRAIC TOPOLOGY TUTORIAL 5

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**Problem 1.** Let  $S^1$  be identified with  $I/\partial I = [0, 1]/\{0, 1\}$ . Show that  $S^1$  is a co- $H$ -group under the comultiplication  $\mu'$  defined by

$$\mu'(t) = \begin{cases} (2t, *) & 0 \leq t \leq 1/2 \\ (*, 2t - 1) & 1/2 \leq t \leq 1. \end{cases}$$

and a homotopy inverse  $\nu$  defined by  $\nu(t) = 1 - t$ .

**Problem 2.** Let  $\mu_1$  and  $\mu_2$  be two multiplications on  $Y$  such that  $Y$  is an  $H$ -space under  $\mu_1$  and  $\mu_2$ . Show that

$$\Omega\mu_1 \simeq \Omega\mu_2: \Omega(Y \times Y) \rightarrow \Omega Y.$$

**Problem 3.** Let  $\lambda$  and  $\mu$  be paths in  $X$  from  $x$  to  $y$ . Suppose that  $X$  is simply connected. Then  $\lambda \simeq \mu$ .

**Problem 4.** Let  $X$  and  $Y$  be locally compact Hausdorff pointed spaces. Show that  $\Sigma(X \vee Y)$  is a weak retract of  $\Sigma(X \times Y)$ .

**Problem 5.** Suppose that  $X$  is a locally compact Hausdorff space. Show that  $\Sigma X$  is a retract of  $\Sigma\Omega\Sigma X$ .

**Problem 6.** Let  $Y$  be a locally compact Hausdorff space. Suppose that  $X$  and  $Y$  are co- $H$ -spaces. Show that  $X \wedge Y$  is a homotopy associative and homotopy commutative co- $H$ -space.