INTRODUCTION TO ALGEBRAIC TOPOLOGY
ANSWERS TO TUTORIAL 7

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Problem 1. Construct a space $X$ such that $\pi_1(X)$ is

i) the cyclic group $\mathbb{Z}/n$.
ii) generated by $x, y$ with the relation $x^2 = y^3$.
iii) generated by $x, y, z$ with the relations $x = [y, z], y = [x, z]$ and $z = [x, y]$,
where $[a, b] = a^{-1}b^{-1}ab$ is called the commutator of elements $a, b$.

(Justify your answers.)

Hint. Read Theorem 3.5.7 and Example 3.5.8.  

Problem 3. [Pull-back] Let $p: \tilde{X} \rightarrow X$ be a covering projection and let $f: Y \rightarrow X$ be a map. Let

$$\tilde{Y} = \{(y, \tilde{x}) \in Y \times \tilde{X} | f(y) = p(\tilde{x})\}$$

and let $q: \tilde{Y} \rightarrow Y$ be defined by $q(y, \tilde{x}) = y$. Show that $q$ is a covering projection.

Hint. Let $y \in Y$ and let $U$ be an elementary neighbourhood of $f(y)$. Show that $f^{-1}(U)$ is an elementary neighbourhood of $y$.  

Problems 5 and 6 are from the text book (Czes Kosniowski, A first course in algebraic topology, Cambridge University Press, 1980. pp. 148 Exercise 17.9 (e) and (f)).