INTRODUCTION TO ALGEBRAIC TOPOLOGY
ANSWERS TO TUTORIAL 7

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Problem 1. Construct a space X such that $\pi_1(X)$ is

i) the cyclic group $\mathbb{Z}/n$.
ii) generated by x, y with the relation $x^2 = y^3$.
iii) generated by x, y, z with the relations $x = [y, z], y = [x, z]$ and $z = [x, y]$, where $[a, b] = a^{-1}b^{-1}ab$ is called the *commutator* of elements a, b.

(Justify your answers.)

Hint. Read Theorem 3.5.7 and Example 3.5.8. □

Problem 3.[Pull-back] Let $p\colon \tilde{X} \to X$ be a covering projection and let $f\colon Y \to X$ be a map. Let

$$\tilde{Y} = \{(y, \tilde{x}) \in Y \times \tilde{X} | f(y) = p(\tilde{x})\}$$

and let $q\colon \tilde{Y} \to Y$ be defined by $q(y, \tilde{x}) = y$. Show that q is a covering projection.

Hint. Let $y \in Y$ and let U be an elementary neighbourhood of $f(y)$. Show that $f^{-1}(U)$ is an elementary neighbourhood of y. □

Problems 5 and and **6** are from the text book (Czes Kosniowski, *A first course in algebraic topology,* Cambridge University Press, 1980. pp. 148 Exercise 17.9 (e) and (f)).