

1. Use the ratio test to determine the convergence or divergence of each of the following series.

i) $\sum_{n=1}^{\infty} \frac{(3n)!}{6^n n! (2n)!}$

ii) $\sum_{n=1}^{\infty} a_n$, where $a_1 = 1$, $a_n = 2 \left(1 + \frac{1}{2n}\right)^n a_{n-1}$, $n = 2, 3, \dots$.

2. Use the (simplified) root test to determine the convergence or divergence of each of the following series.

i) $\sum_{n=1}^{\infty} \frac{5n^2 \cdot 3^n}{4^{n+4}}$.

ii) $\sum_{n=1}^{\infty} \frac{3^{2n}}{5^n} \left(1 - \frac{1}{2n}\right)^{n^2}$.

iii) $\frac{1}{4} + \frac{1}{5^2} + \frac{1}{4^3} + \frac{1}{5^4} + \frac{1}{4^5} + \frac{1}{5^6} + \frac{1}{4^7} + \frac{1}{5^8} + \dots$.

3. Determine the convergence or divergence of each of the following series. Justify your answers.

i) $\sum_{n=1}^{\infty} (\sqrt{2n+2} - \sqrt{n})$.

ii) $\sum_{n=1}^{\infty} \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{n!} \cdot \frac{2^n}{5^n}$.

iii) $\sum_{n=1}^{\infty} \frac{\ln n}{n^{1.2}}$.

iv) $\sum_{n=1}^{\infty} \left(\frac{n}{n+2}\right)^{n^2}$.

v) $\sum_{n=1}^{\infty} \frac{1}{(\ln n)^3}$.

vi) $\sum_{n=1}^{\infty} \left(\frac{4}{9} + \frac{n^3}{3^n}\right)^{\frac{n}{2}}$.

4. Show that the series $\sum_{n=1}^{\infty} (-1)^n \frac{\cos n}{2^n}$ is absolutely convergent.

5. Consider the series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{\ln n}{\sqrt{n}}$.

i) Use the alternating series test to show that the series is convergent.

ii) Using part i) or otherwise, show that the series is conditionally convergent.

6. For each of the following series, determine whether the series is absolutely convergent, conditionally convergent or divergent. Justify your answers.

i) $\sum_{n=1}^{\infty} (-1)^n \frac{3}{2n+1}$.

ii) $\sum_{n=1}^{\infty} (-1)^n \frac{n}{4n+3}$.

iii) $\sum_{n=1}^{\infty} (-1)^n \left(\frac{1+2n}{3+4n}\right)^n$.

iv) $\sum_{n=2}^{\infty} (-1)^{n+1} \frac{1}{n \ln n}$.

Some suggested answers:

1. i) divergent.
1. ii) convergent.
2. i) convergent.
2. ii) divergent.
2. iii) convergent.
3. i) divergent.
3. ii) convergent.
3. iii) convergent.
3. iv) convergent.
3. v) divergent.
3. vi) convergent.
6. i) conditionally convergent.
6. ii) divergent.
6. iii) absolutely convergent.
6. iv) conditionally convergent.