1. 'Classify' the following differential equations, i.e., determine whether the following equations are homogeneous/non-homogeneous, linear/non-linear, 1st order/2nd order/$\cdots$, ordinary differential equations:

a) $t\dfrac{dy}{dt} + y\sin t = 0;$

b) $(1+t^2)\dfrac{dy}{dt} + 2ty + 3 = 0;$

c) $\dfrac{dy}{dt} + \dfrac{d^3y}{dt^3}\dfrac{dy}{dt} = 3t;$

d) $4\dfrac{d^2y}{dt^2} + 5\dfrac{dy}{dt} + 6y = e^t + \sin 2t;$

e) $u\dfrac{d^4u}{dt^4} + \left(\dfrac{du}{dt}\right)^2 = 0.$

2. Find the general solution of the following differential equations:

a) $\dfrac{dy}{dt} + y\sin t = 0;$

b) $(1+t^2)\dfrac{dy}{dt} + ty = (1+t^2)^{3/2}$

3. Solve the following initial/boundary value problems, and express your answers in the form "y=y(t)".

a) $\dfrac{dy}{dt} + 4ty = 2te^{2t^2}, \quad y(1) = 5.$

b) $y\dfrac{dy}{dt} - 2ty^4 = 0, \quad y(0) = \dfrac{1}{2}.$

4. Find the general solution of the following differential equations: $\dfrac{dy}{dt} = \dfrac{y}{t} - \dfrac{y^3}{t^3}.$

5. Find the the general solution of the following differential equations:

a) $\dfrac{d^2y}{dt^2} - 7\dfrac{dy}{dt} + 10y = 0;$

b) $\dfrac{d^2y}{dt^2} + 6\dfrac{dy}{dt} + 9y = 0;$

c) $\dfrac{d^2y}{dt^2} + 2\dfrac{dy}{dt} + 3y = 0.$

6. Solve the following initial value problem:

$$\frac{d^2y}{dt^2} - 3\frac{dy}{dt} - 4y = 0; \quad y(0) = 4, \quad y'(0) = 0.$$

7. Use the method of undetermined coefficients to find a particular solution of the following differential equations:

$$y'' + 3y = 6t^3 - 6.$$

8. Find the general solution of the following differential equations:

i) $y'' - 3y' + 2y = 3e^{2t} + e^{3t}$.
ii) $y'' + 4y = \cos 2t$.

9. Solve the following initial value problem:

$$y'' + 4y' + 3y = 2e^{-3t}; \quad y(0) = 0, \quad y'(0) = 3.$$

10. A tank containing S_0 lb of salt dissolved in 200 gallons of water. Starting at time $t = 0$, water containg $\frac{1}{2}$ lb of salt per gallon enters the tank at rate of 4 gal/min, and well stirred solution leaves the tank at the same rate. Find the concentration of salt in the tank at any time $t > 0$.

Some suggested answers:
1 a) homogeneous linear 1st order ODE
1 b) non-homogeneous linear 1st ODE
1 c) non-homogeneous non-linear 3rd order ODE
1 d) non-homogeneous linear 2nd order ODE
1 e) homogeneous non-linear 4-th order ODE.
2 a) $y = Ce^{\cos t}$
2 b) $y = \dfrac{1}{\sqrt{1+t^2}}\left(t + \dfrac{t^3}{3} + C\right)$
3 a) $y = e^{-2t^2}\left(t^2 + 5e^2 - 1\right)$
3 b) $y = \dfrac{1}{\sqrt[3]{8 - 3\ln(1+t^2)}}$
4 $y = \dfrac{t}{\sqrt{2(\ln|t| + C)}}$
5 a) $y(t) = Ae^{2t} + Be^{5t}$
5 b) $y(t) = (At + B)e^{-3t}$
5 c) $y(t) = Ae^{-t}\cos\sqrt{2}t + Be^{-t}\sin\sqrt{2}t$
6. $y(t) = \dfrac{4}{5}e^{4t} + \dfrac{16}{5}e^{-t}$
7. $y_p(t) = 2t^3 - 4t - 2$
8 a) $y(t) = Ae^t + Be^{2t} + 3te^{2t} + \dfrac{1}{2}e^{3t}$
8 b) $y(t) = A\cos 2t + B\sin 2t + \dfrac{t}{4}\sin 2t$
9. $y(t) = 2e^{-t} - 2e^{-3t} - te^{-3t}$
10. the concentration $c(t)$ of salt in the tank is given by

$$c(t) = \frac{S_0}{200}e^{-0.02t} + \frac{1}{2}\left(1 - e^{-0.02t}\right).$$