

A Question About the tangent bundle of spheres (Jie Wu)

Let $E(2n)$ denote the Stiefel manifold $V_{2n+1,2}$, the manifold of unit tangent vectors on S^{2n} . Let $i: S^{2n-1} \rightarrow E(2n)$ be the inclusion of a fibre and let $*$ be the base-point of S^{2n-1} .

Problem:

Does there exist a continuous map

$$f: S^{2n-1} \times S^{2n-1} \rightarrow E(2n)$$

such that

$$f|_{S^{2n-1} \times *} = f|_{* \times S^{2n-1}} = i: S^{2n-1} \rightarrow E(2n)?$$

Remarks:

- 1) The positive answer is equivalent to that the Whitehead square ω_{2n-1} is divisible by 2.
- 2) The answer is NO if $2n$ is not a power of 2.
- 3) The answer is YES if $2n = 2, 4, 8, 16, 32, 64$.
- 4) The answer is UNKNOWN if $2n = 128$.
- 5) The answer is NO if $2n = 2^k$ with $k \geq 8$. (**This follows from the recent solution to the Kervaire invariant problem given by Mike Hopkins, Doug Ravenel and Mike Hill**).