

1. a)  $8 - 10i$ ; b)  $\frac{-253-204i}{65^2}$ ; c)  $-\frac{3\pi}{4}$ ; d)  $5\sqrt{26}$ ; e)  $e^3\sqrt{3} + e^3i$ .
2. a)  $\pm 2, \pm 2i$ ; b)  $1, 1 \pm \sqrt{3}i$ ; c)  $\frac{1}{e^{2k\pi i/5} - 1}, k = 1, 2, 3, 4$ .
3. a) ellipse with foci at  $\pm 1$ ; b) circle with center at  $1 + i$ ; c) vertical line  $x = 2$
4. a)  $-8 - 8i$ ; b)  $2^{-11}(-1 + \sqrt{3}i)$ ; c)  $\pm \sqrt[4]{2}e^{3\pi i/8}$ ; d)  $e^{(\frac{\pi}{4} + 2k\pi)i/8}, k = 0, 1, 2, \dots, 8$ .
- 5.

$$\arg(z^n) = \arg z + \arg z + \dots + \arg z = n \operatorname{Arg} z + 2k\pi i$$

for  $k \in \mathbf{Z}$ .

6. a)

$$\left| \frac{1 - z^n}{1 - z} \right| = \frac{|1 - z^n|}{|1 - z|} = \frac{\sqrt{(1 - \cos(n\theta))^2 + \sin^2(n\theta)}}{\sqrt{(1 - \cos \theta)^2 + \sin^2 \theta}} = \left| \frac{\sin(n\theta/2)}{\sin(\theta/2)} \right|$$

b)

$$\left| \frac{\sin(n\theta/2)}{\sin(\theta/2)} \right| = \left| \frac{1 - z^n}{1 - z} \right| = |1 + z + z^2 + \dots + z^{n-1}| \leq 1 + |z| + |z|^2 + \dots + |z|^{n-1} = n$$

7. Hint:

$$c^n - 1 = (c^{n-1} + c^{n-2} + \dots + c + 1)(c - 1).$$

8. Open sets: b), c), f)

Domains: b) and c)

Bounded sets: a), c)

Boundary: a)  $|z - 1 + i| = 3$ ; b)  $|\operatorname{Arg} z| = \pi/4$ ; c)  $\{2\} \cup \{|z - 2| = 3\}$ ; d)  $\operatorname{Im} z = \pm 1$ ; e)  $|z| = 2$ ; f)  $\operatorname{Re} z = \pm 1$ .

Regions: a) b) c) d) e)

Closed regions: a) e)