1. a) $(e^{3x}+e^{-3x})\cos 3y+i(e^{3x}-e^{-3x})\sin 3y$. b) $\frac{x}{x^2+y^2}-\frac{yi}{x^2+y^2}$. c) $\frac{x}{x^2+(y-1)^2}+\frac{(1-y)i}{x^2+(y-1)^2}$.

2. a) domain of definition: **C**; range $\mathbf{C}-\{0\}$. **c)** the circle $|w|=e$; **d)** half-line $w \not= 0$ and arg$w=\pi/4$; **e)** infinite sector $w \not= 0$ and $0 \le rgw \le \pi/4$.

3. a) $6i$; **b)** $-1/2$; **c)** $-\frac{\pi}{2}+i$ **d)** 1 **e)** π **f)** does not exist

4. Check that the Cauchy-Riemann equations do not hold. Or prove directly by using definition.

5. a)

$$\frac{2z(iz^3+2z+\pi)-(z^2-9)(3iz^2+2)}{(iz^3+2z+\pi)^2}$$

b)

$$2(z+2)(z^2+iz+1)^{-4}-4(z+2)^2(z^2+iz+1)^{-5}(2z+i)$$

6. a) $z=2-3i$ **b)** $z=1,1/2$

7. a) $f(z)=\frac{z}{\bar{z}+2}$ is nowhere analytic; (Note: $f(z)$ is not differentiable at each point in $\mathbf{C}-\{0\}$ and $f(z)$ is differentiable at $z=0$.) **b)** $x^2-y^2+2xyi=z^2$ is entire **c)** $\left(x+\frac{x}{x^2+y^2}\right)+i\left(y-\frac{y}{x^2+y^2}\right)=z+\frac{\bar{z}}{z\bar{z}}=z+\frac{1}{z}$ is analytic in $\mathbf{C}-\{0\}$.

8.

$$\frac{\partial u}{\partial x}(0,0)=\lim_{\Delta x\to 0}\frac{u(\Delta x,0)-u(0,0)}{\Delta x}=0.$$

Similarly,

$$\frac{\partial u}{\partial y}(0,0)=\frac{\partial v}{\partial x}(0,0)=\frac{\partial v}{\partial x)(0,0)=0.}$$

9. Since the first derivatives of $u(x,y)=x^3+3xy^2-3x$ and $v(x,y)=y^3+3x^2y-3y$ are continuous, $h(z)$ is differentiable at $z=x+yi$ if and only if the Cauchy-Riemann equations hold. Thus $h(z)$ is differentiable in

$\{(x,y)|\ xy=0\}$ and is NOT diffrentiable in $\{(x,y)|xy\neq 0\}$

$h(z)$ is nowhere analytic because $h(z)$ is not differentiable at some points in any neighbourhood of z for any z.