1. Show that the function $f(z) = e^{x^2-y^2}[\cos(2xy) + i\sin(2xy)]$ is entire, and find its derivative.

2. If u and v are expressed in terms of polar coordinates (r, θ), show that the Cauchy-Riemann equations can be written in the form

$$\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta} \qquad \frac{\partial v}{\partial r} = -\frac{1}{r}\frac{\partial u}{\partial \theta}.$$

3. Find the most general harmonic polynomial of the form $ax^2 + bxy + cy^2$.

4. Suppose that the function u and v are harmonic in a domain D.

a) Is the sum $u + v$ necessarily harmonic in D?

b) Is the product uv necessarily harmonic in D?

c) Is $\partial u/\partial x$ harmonic in D?

5. Verify that each given function u is harmonic in some domain and then find a harmonic conjugate of u

a) $u = \sin x \cosh y$

b) $u = \ln|z|$ for $\operatorname{Re} z > 0$

c) $u = \operatorname{Im}(e^{z^2})$

d) $u = y/(x^2 + y^2)$.

6. Sketch the families of level curves of the component functions u and v when

a) $f(z) = z^2$

b) $f(z) = \frac{z-1}{z+1}$

7.Find a function $\phi(x, y)$ that is harmonic in the region of the first quadrant between the curves $xy = 2$ and $xy = 4$ and takes the value 1 on the lower edge and value 3 on the upper edge. [Hint: Begin by considering z^2.]

8. Find a function $\phi(x, y)$ that is harmonic in the upper half-plane $\operatorname{Im} z > 0$ and continuous on $\operatorname{Im} z \geq 0$ such that

a) $\phi(x, 0) = x^2 + 5x + 1$ for all x.

b) $\phi(x, 0) = 2x^3/(x^2 + 4)$ for all x.