

ORDERS OF WHITEHEAD PRODUCTS OF ι_n WITH $\alpha \in \pi_{n+k}^n$ ($n \geq k + 2, k \leq 24$)

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This is partly jointed work with M. Golasiński. The following result is obtained by using mainly the results of Mahowald, Mimura, Nomura, Oda, Toda.

Theorem 0.1 *The order of $[\iota_n, \alpha]$ for $n \equiv i \pmod{8}$ is given as follows except as otherwise noted.*

$\alpha \setminus i$	0	1	2	3	4	5	6	7
η	2	2	2	0	2	2	2	0
η^2	2	2	0	0	2	2	0	0
ν	8	2	4	2	8	2, $\neq 2^i - 3$ 0, $= 2^i - 3$	4	0
ν^2	2	2	2	2, $\neq 2^i - 5$ 0, $= 2^i - 5$	0	0	2	0
σ	16	2	16	2	16	2	16	2, 7(16) 0, 15(16)
$\eta\sigma$	2	2	2	0	2	2	0, $\neq 22(32) \geq 54$ 2, $\equiv 22(32) \geq 54$	0
ε	2	2	0	0	2	2	2, $\neq 22(32) \geq 54$ 0, $\equiv 22(32) \geq 54$	0
$\bar{\nu}$	2	2	2	0	2	2	2	0
$\eta^2\sigma$	2	2, $\neq 2^i - 7$ 0, $= 2^i - 7$	0	0	2	0, $\neq 53(64)$ 2, $\equiv 53(64)$ ≥ 117	0	0
$\eta\varepsilon$	2	0	0	0	2	0, $\neq 53(64)$ 2, $\equiv 53(64)$ ≥ 117	0	0
ν^3	2	2, $\neq 2^i - 7$ 0, $= 2^i - 7$	0	0	0	0	0	0
μ	2	2	2	0	2	2	2	0
$\eta\mu$	2	2	0	0	2	2	0	0
ζ	8	0	4	0, $\neq 115(128)$ 2, $\equiv 115(128)$ ≥ 243	8	0	4	0
σ^2	2, 0(16)	0, 9(16)	2	0, 11(16)	2		2	0, 15(16)
κ	2	2	2	2	2	2	2	0

$\alpha \setminus i$	0	1	2	3	4	5	6	7
$\eta\kappa$	2	0	0	0	2	2	0	0
ρ	32	2	32	2	32	2	32	0, $\neq 239(256)$ 2, $\equiv 239(256)$ ≥ 495
$\eta\rho$	2	2	2	0	2	2	0, $\neq 494(2^9)$ 2, $\equiv 494(2^9)$ ≥ 1006	0
η^*	2	2	2	0	2	2		0
$\eta^2\rho$	2	2	0	0	2	0, $\neq 1005(2^{10})$ 2, $\equiv 1005(2^{10})$ ≥ 2029	0	0
$\eta\eta^*$	2	2	0	0	2		0	0
$\nu\kappa$	2	0	2	2	2	0	0	0
$\bar{\mu}$	2	2	2	0	2	2	2	0
$\eta\bar{\mu}$	2	2	0	0	2	2	0	0
ν^*	8	2	4	2	4 or 8		4	0
$\bar{\zeta}$	8	0	4	0, $\neq 2027(2^{11})$ 2, $\equiv 2027(2^{11})$ ≥ 4075	8	0	4	0
$\bar{\sigma}$	2	2	2	2		0		0
$\bar{\kappa}$	8	2	4 or 8	2	4	2	4	0
σ^3	0, 8(16)	0, 9(16)	2	0, 11(16)	0	0	2	0
$\eta\bar{\kappa}$	2	2	2	0	2	2		0
$\eta^2\bar{\kappa}$	2		0	0	2		0	0
$\nu\bar{\sigma}$					0	0	0	0
$\eta^*\sigma$	2	2	0	0	2	2	0, $\equiv 6(16) \geq 22$	0
$\nu\bar{\kappa}$	4 or 8		4	2	4	0, $2^i - 3$	4	0
$\bar{\rho}$	16	2	16	2	16	2	16	0, $\neq 4071(2^{12})$ 2, $\equiv 4071(2^{12})$ ≥ 9167
$\eta\eta^*\sigma$	2		0	0	2		0	0
$\eta\bar{\rho}$	2	2	2	0	2	2	0, $\neq 9166(2^{13})$ 2, $\equiv 9166(2^{13})$ ≥ 18358	0

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