Let $x_0 \in D$. One considers the sequence $\{F^n(x_0)\}_{n=0}^\infty$.

\[
\| F^n(x_0) - F^{n-1}(x_0) \| = \| F(F^{n-1}(x_0)) - F(F^{n-2}(x_0)) \| \\
\leq \alpha \| F^{n-1}(x_0) - F^{n-2}(x_0) \| = \alpha \| F(F^{n-2}(x_0)) - F(F^{n-3}(x_0)) \| \\
\leq \alpha^2 \| F^{n-2}(x_0) - F^{n-3}(x_0) \| \leq \alpha^n \| F(x_0) - x_0 \|.
\]

Since $\alpha < 1$, then $\lim_{n \to \infty} n \geq n$

\[
\| F^n(x_0) - F^{n-1}(x_0) \| \leq \frac{\alpha^{n-1}}{1-\alpha} \| F(x_0) - x_0 \|.
\]

Thus, $\{ F^n(x_0) \}_{n=0}^\infty$ is a Cauchy sequence.

By the property that $D$ is a closed set, the Cauchy sequence $\{ F^n(x_0) \}_{n=0}^\infty$ converges.

Let $x = \lim_{n \to \infty} F^n(x_0) \in D$.

\[
x = \lim_{n \to \infty} F(F^{n-1}(x_0)) = F(\lim_{n \to \infty} F^{n-1}(x_0)) = F(x).
\]

Here we will use that $F$ is a continuous function.
Problem 2.

Suppose \( \frac{\partial u}{\partial t} + c \frac{\partial u}{\partial x} = 0 \)
\[ u(x,0) = f(x). \]

The characteristic curve \((x(s), t(s))\)
\[ \begin{cases} \frac{dx}{ds} = 1, & \frac{dT}{ds} = c, \\ T(0) = t, & \text{for } x(t) = x \end{cases} \]

One has that
\[ \frac{d}{dt} u(x(s), t(s)) = \frac{\partial u}{\partial x} \frac{dx}{ds} + \frac{\partial u}{\partial t} \frac{dT}{ds} = \frac{\partial u(x(s), t(s))}{\partial x} + c \frac{\partial u(x(s), t(s))}{\partial t} = \frac{\partial u(x(s)), t(s))}{\partial x} = 0 \]
\[ \Rightarrow u(x(s), t(s)) = u(x(0), t(0)) \quad \forall s \in \mathbb{R}. \]

Substitute \( x(s) = x + c(s-t) \) into (1)
\[ \begin{cases} x(s) = x + c(s-t) \\ T(s) = s \end{cases} \]

to yield that
\[ u(x,t) = u(x(t), T(t)) = u(x(0), T(0)) = u(x - ct, 0) = f(x - ct) \]