\[(n, \Lambda, x)\sigma = \lambda n(n, \Lambda, x)q + \lambda n(n, \Lambda, x)\nu\]

Quasi-linear equation

\[0 = (\underbrace{n, \ldots, \Lambda, x}_{\lambda n, \ldots, \Lambda, x}) f\]

First order equation
Solution (Integral Surface) for

\[(\nabla'x)n = z\]

\[\mathbf{v} = (n, \mathbf{x})(q + \nabla n)\]

\[
\begin{align*}
(\mathbf{\nabla}'x)n &= z \\
\quad : (n, \mathbf{\nabla}'x)c &= \mathbf{\nabla} n(n, \mathbf{\nabla}'x)q + \mathbf{\nabla} n(n, \mathbf{\nabla}'x)v
\end{align*}
\]
Vector field

\[ \mathbf{V}(x, y, z) = (U_x(x, y, z), U_y(x, y, z), U_z(x, y, z)) \]

\[ (a(x, y, z), b(x, y, z), c(x, y, z)) \]

\[ (a(x, y, z), b(x, y, z), c(x, y, z)) \cdot (U_x, U_y, U_z) = 0 \]
Theorem. Let the point $P = (x, y, z)$ lie on the integral surface $S$ through $P$. Let $\gamma$ be the integral curve through $P$. Then $\gamma$ lies completely in $S$.
\[ A \leq (s_1)(n-z) \]
The Cauchy Problem for the Quasi-linear Equation

\[(zu_{x}y_{y}) = (1, x, y, \text{Non-characteristic})\]

\[\forall s \in \mathbb{R}, (s)\eta = ((s)\alpha, (s)\mu)n\]

\[\{R \supset s \mid ((s)\eta, (s)\alpha, (s)\mu)\} : 1\]

The Cauchy Problem for the Quasi-linear Equation
\[(x) \eta = (0', x) n\]

\[0 = x n c + \xi n\]

Example
\[
\left\{(s_1, s_2, \ldots, s_{n-1}) = \emptyset \cap \bigcap_{k=1,2,\ldots, n-1} \mathcal{X} \right\} \cap \mathcal{X} \quad n = \mathbb{Z}
\]

\[
(0 \neq \mathcal{A}) \quad n \mathcal{A} = \sum_{u=1}^{n} n^{y_x} x
\]

Example
\[ Z = \left( x_n, \frac{x_n^2}{2}, \ldots, \frac{x_n^k}{k} \right) \]

\[ X_k = x_n \subseteq S \Rightarrow X_k = X \]

\[ X_n = t \Rightarrow x_n \in e \]

\[ (\forall x_n = t) \]

\[ x_n \in e / e \]

\[ \{ x_n(s_0, \ldots, s_{n-1}) = e \} \]
In order to apply

\[ \begin{align*}
\{ y = x \mid y \in U \} & \quad \text{s.t.} \quad z = x \\
0 = x + y & \quad 1 = \frac{z}{x}
\end{align*} \]

\( x \in \{ y \mid y \in U \} \quad \Rightarrow \quad s + t = x \quad \leq \quad s = 0 \quad \Rightarrow \quad z = \frac{2e}{xe} \)

\( (x)\eta = (0, x)\eta \quad \Rightarrow \quad 0 = xnn + \xi n \)
\[
\frac{x+y}{x} = n \iff y = \frac{x}{n} (1 - n y) \]

Then one can apply the implicit function theory

\[ \exists n \neq 0 \text{ and } A > 0, \]

\[ (y - x - y) + 1 = 1 - y (-y - y) \]

The implicit function theorem.
\( o = x \nu \eta + \nu \eta \)
\frac{\sqrt{\frac{x}{4}}}{x} = \frac{1}{16} e^{\int \frac{1}{x} \frac{1}{4} (h(x)^{1/4} + (h(x)^{1/4})^x) \, dx} = 0 = \frac{1}{x^{1/4} h(x)^{1/4}} = 0 \\
(1, 0) \neq 0 \\
Implicit Function Theorem
Let \( u \) be a solution of the Cauchy problem in the close unit disc \( \Omega \) in the \( x-y \)-plane.

Let \( \Omega \) be a solution of the Cauchy problem in the close unit disc \( \Omega \) in the \( x-y \)-plane.
2. Show that the solution of the quasi-linear equation

\[ u_x + hu_y = 0 \]

with initial condition \((y,0) = (x)\) is given implicitly by

\[ u = \psi \left( \frac{y}{x} \right) \]

unless \(\psi \) is a non-decreasing function of \(x\).

Show that the solution becomes singular for some positive time \(T\)

\[ (\lambda(n)v - x)y = n \]

with initial condition \((x)y = (0,x)n\) given implicitly by

\[ 0 = \frac{\partial}{\partial x} n(n)v + \frac{\partial}{\partial n} n \]

2. Show that the solution of the quasi-linear equation
General First-Order Equation for a Function of two variables

\[ F(x,y,u,u_x,u_y) = 0. \]
\[ \frac{dg}{dp} = \frac{d(x-x_0)}{dp} + \frac{d(y-y_0)}{dp} = 0 \]

Monge Cone.

\[ p(x-x_0) + y(y-y_0) \cdot \frac{dp}{dy} = 0 \]

The envelope of \( p(x-x_0) + y(y-y_0) \) is called the...
Homework

I. For the equation $n = \frac{\partial}{\partial z}$ with $(x, 0) = 2x_3/3$, find a solution.

II. For the equation $n = \frac{\partial}{\partial z}$ with $(x, 0) = 2x_3/3$, find a solution.

Due Week 4