1. Player 1 and player 2 have been willed equal shares of an estate consisting of $200,000 cash and 100 acres of farmland. Player 1 has a sentimental attachment to the land and values it at $3,000 per acre, whereas player 2 has no such attachment and values it at $1,000 per acre. Assume that their payoff functions are linear in money and land at these rates: 

\[ u_i = x_i + v_i y_i \]

if player \( i \) receives \( x_i \) dollars of cash and \( y_i \) acres of land. The players may reach an agreement on dividing the land and money so as to maximize their payoffs. If they fail to reach agreement they divide the land and money equally.

(i) Carefully draw the bargaining set and label the disagreement point.

(ii) Find the Nash bargaining solution.

2. Let \( \delta_i = v(N) - v(N \setminus \{i\}) \), \( i = 1, 2, \ldots, n \), for a cooperative game \((N, v)\). Show that the core is empty if \( \sum_{i=1}^{n} \delta_i < v(N) \).

3. Consider the following three-person game:

\[
\begin{align*}
    v(\emptyset) &= 0, & v(\{1\}) &= 0.2, & v(\{2\}) &= v(\{3\}) = 0, \\
    v(\{1,2\}) &= 1.5, & v(\{1,3\}) &= 1.6, & v(\{2,3\}) &= 1.8, \\
    v(\{1,2,3\}) &= 2.
\end{align*}
\]

(a) Find the core of this game.

(b) Find the Shapley value of this game.

(c) Find an imputation dominating the imputation \((1, 1/2, 1/2)\).

4. The managing board of a corporation consists of three stock-holders who have respectively 20, 30 and 50 shares of stock and the chairman who has no shares. Any decision can be settled by approval of board members holding a simple majority of the shares and the chairman can decide tie votes. Thus, we define a characteristic function as follows. It has value 1 if a coalition holds \( > 50 \) shares or holds \( \geq 50 \) shares and has the chairman in the coalition. It has value 0 otherwise. Find the Shapley value.
5. Three doctors have banded together to form a joint practice: the Port Charles Trio. The overhead for the practice is $40,000 per year. Each doctor brings in annual revenues and incurs annual variable costs as follows: doctor 1 – $155,000 in revenue, $40,000 in variable cost; doctor 2 – $160,000 in revenue, $35,000 in variable cost; doctor 3 – $140,000 in revenue, $38,000 in variable cost.

The Port Charles Trio wants to use game theory to determine how much each doctor should be paid. Determine the relevant characteristic function and show that the core of the game consists of an infinite number of points. Also determine the Shapley value of the game. Does the Shapley value give a reasonable division of the practice’s profits?

6. The management committee of an association consists of a president, a secretary, a treasurer and two committee members. It requires three votes including one from the president, one from the secretary or treasurer to approve a proposal. Determine the voting power of each member in the committee.

7. Consider an n-person game in which the only winning coalitions are those coalitions containing player 1 and at least one other player. If a winning coalition receives a reward of $1, find the core and the Shapley value of the game.

8. Find the Shapley values of the game with $N = \{1, 2\}$ and the characteristic function $v$. Now consider the bargaining game where $H = I(N, v)$ and $d = (v(\{1\}), v(\{2\}))$. Find the bargaining solution of the game $(H, d)$. 