Key words:
- Classical groups, invariant distributions, local theta correspondence.
- Unitary representations, models of representations.

**Figure 1.** Invariant distributions of classical groups

\[
\begin{align*}
U(p) \times U(q) & \quad \text{O}(p, q) \\
O(q) \quad \text{U}(p, q) & \quad \text{Sp}(2n, \mathbb{R}) \times \text{Sp}(2n, \mathbb{R}) \\
& \quad \text{U}(n)
\end{align*}
\]

**Figure 2.** On certain small representations of indefinite orthogonal groups
[Reference: Represent. Theory 1 (1997); with J.-S. Huang]

\[
O(p, q) \curvearrowright \text{NullReg} \ (\subset \text{M}_{p+q,n}(\mathbb{R})) \rightsou equivalent \pi_n
\]

\pi_n \text{ quantizes certain small nilpotent orbit } \mathcal{O}_n, \text{ for } n \leq \frac{1}{2} \min(p, q).

**Figure 3.** Degenerate principal series and local theta correspondence
**Figure 4.** Transfer of unitary representations (between real forms)
[Reference: Asian J. Math. 8 (2004); with N. Wallach]

\[
\begin{array}{c}
g \supset \sigma' \\
\sigma \\
t_c \cap g' \cap k' \supset \tau_c \\
t_c \cap g' \cap k \\
\end{array}
\]

**Figure 5.** Theta lifting and associated cycles

\[
\begin{array}{ccc}
V & \overset{\Xi \times_Y Z}{\longrightarrow} & Z \\
\downarrow{H} & & \downarrow{H} \\
X & \overset{\Xi}{\longrightarrow} & Y
\end{array}
\]

**Figure 6.** Uniqueness of Bessel models
[Reference: GAFA 20 (2010); with D. Jiang and B. Sun]

\[
\forall \pi \in \text{Irr}(G) \text{ and } \pi_0 \in \text{Irr}(G_0) : \\
\dim \text{Hom}_{S_r}(\pi \hat{\otimes} \pi_0, \chi_{S_r}) \leq 1,
\]

where \( S_r = G_0 \ltimes N_{S_r} \) is the \( r \)-th Bessel subgroup of \( G \).
Figure 7. Multiplicity one theorems (for classical spherical pairs)
[Reference: Annals of Math. 175 (2012); with B. Sun]

\[ \forall \pi \in \text{Irr}(G) \text{ and } \pi' \in \text{Irr}(G') : \]
\[ \dim \text{Hom}_{G'}(\pi, \pi') \leq 1. \]

Figure 8. Theta lifting of generalized Whittaker models
[Reference: GAFA 24 (2014); with R. Gomez]

\[ \text{Hom}(V, \tilde{V}) \]
\[ \varphi \]
\[ \mathcal{O} \subset \mathfrak{g} \]
\[ \tilde{\mathfrak{g}} \subset \tilde{\mathcal{O}} \]

\[ \text{Wh}_{\tilde{\mathcal{O}}}(\Theta(\pi)) \cong \text{Wh}_{\mathcal{O}}(\tilde{\pi}), \ \forall \pi \in \text{Irr}(G). \]

Figure 9. Conservation relations for local theta correspondence
[Reference: Jour. Amer. Math. Soc. 28 (2015); with B. Sun]

\[ \forall \pi \in \text{Irr}(\bar{G}(U)) : \]
\[ n_{t_1}(\pi) + n_{t_2}(\pi) = 2 \dim U + d_{D,\epsilon}, \]

where \( d_{D,\epsilon} = 0, 1, 2, 3, 4, \) depending on \( D \) and \( \epsilon. \)