Workshop on Representation Theory

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National University of Singapore

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NUS National University of Singapore | Department of Mathematics, Faculty of Science

Programme (Day 1)

9.30am – 10.30am	**Dirac cohomology of elliptic representations** *Jing-Song Huang* *Hong Kong University of Science and Technology, Hong Kong*
10.30am – 10.45pm	Break @ Mathematics Department Lounge
10.45am – 11.45am	**Dirac operators and duality** *Fuhai Zhu* *Nankai University, China*
11.45am – 2.00pm	Lunch
2.00pm – 3.00pm	**On a conjecture of Jiang on Fourier coefficients and Arthur parameters for classical groups** *Baiying Liu* *University of Utah, USA*
3.00pm – 3.30pm	Break @ Mathematics Department Lounge
3.30pm – 4.30pm	**Bessel models and a conjecture of Guo and Jacquet** *Masaaki Furusawa* *Osaka City University, Japan*
5.30pm	Excursion + Dinner

All talks are held in S17, Room 04-04 (level 4, Seminar Room 3)

For directions to S17, refer to http://ww1.math.nus.edu.sg/contactus.aspx

Programme (Day 2)

9.30am – 10.30am	**Models of real reductive Lie groups** *Meng-Kiat Chuah* *National Tsing Hua University, Taiwan*
10.30am – 10.45pm	Break @ Mathematics Department Lounge
10.45am – 11.45am	**Degenerate Whittaker models for real reductive groups** *Dmitry Gourevitch* *Weizmann Institute of Science, Israel*
11.45am – 2.00pm	Lunch
2.00pm – 3.00pm	**On the set of maximal nilpotent supports of supercuspidal representations** *Yujun Qin* *East China Normal University, China*
3.00pm – 3.30pm	Break @ Mathematics Department Lounge
3.30pm – 4.30pm	**On the residual spectrum of a quasi-split group of type D4** *Jing Feng Lau* *National University of Singapore, Singapore*
6.00pm	Workshop Dinner

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Programme (Day 3)

9.30am – 10.30am	**A quick survey on monomial ideals** *Takayuki Hibi* *Osaka University, Japan*
10.30am – 10.45pm	Break @ Mathematics Department Lounge
10.45am – 11.45am	**The Littlewood-Richardson triangles and highest weight vectors** *Soo Teck Lee* *National University of Singapore, Singapore*
11.45am – 2.00pm	Lunch
2.00pm – 3.00pm	**Theta correspondence between supercuspidal representations** *Jiajun Ma* *Ben-Gurion University, Israel*
3.00pm – 3.30pm	Break @ Mathematics Department Lounge
3.30pm – 4.30pm	**A proof of the Howe Duality Conjecture** *Wee Teck Gan* *National University of Singapore, Singapore*
5.30pm	Excursion + Dinner

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Abstracts (Day 1)

Dirac cohomology of elliptic representations
Jing-Song Huang

An elliptic representation is a representation whose global character does not vanish on the set of regular elliptic elements. The set of irreducible elliptic representations are exactly the set of irreducible representations with nonzero Dirac index. We describe a classification of the equivalence classes of irreducible elliptic representations with regular infinitesimal characters. Then we formulate some questions and conjectures for applications in harmonic analysis.

Dirac operators and duality
Fuhai Zhu

It is well-known that nonzero Dirac cohomology of an irreducible unitary representation determines the infinitesimal character of such representation. But in most cases, unitary representations have trivial Dirac cohomology. In this talk, I will explain the duality between representations of compact Lie groups and those of Lie superalgebras based on the action of Dirac operators on any given irreducible unitary representation.

On a conjecture of Jiang on Fourier coefficients and Arthur parameters for classical groups
Baiying Liu

In 2012, Jiang made a conjecture on relations between structures of Fourier coefficients of automorphic representations in automorphic L^2-packets and structures of Arthur parameters for classical groups. In this talk, I will review this conjecture and present some recent progress towards it. This is a joint work in progress with Dihua Jiang.

Bessel models and a conjecture of Guo and Jacquet
Masaaki Furusawa

Let E be a quadratic extension of a number field F and D a quaternion algebra over F containing E. Let π_D be a cuspidal automorphic representation of GL(n,D) and π its Jacquet-Langlands transfer to GL(2n,F). Guo and Jacquet conjectured that if π_D is distinguished by $GL(n,E)$, then π is symplectic and $L(1/2,\pi_E)\ne 0$, where π_E is the base change of π to E. When n is odd, Guo and Jacquet conjectured also a converse. The converse does not always hold when n is even, but we conjecture it holds if and only if the corresponding generic representation of the split special orthogonal group SO(2n+1) has a special Bessel E-model. When n=2, we deduce part of our conjecture by relating E-Bessel periods on SO(5) with GL(2,E)-periods on GL(2,D). This is a joint work with Kimball Martin (University of Oklahoma).

Abstracts (Day 2)

Models of real reductive Lie groups
Meng-Kiat Chuah

We construct unitary representations of real reductive Lie groups by symplectic geometric method. These representations are multiplicity-free. They consist of series of representations parametrized by integral weights of Cartan subalgebras, such as the discrete series and principal series. The occurrences of these parameters in the unitary representations are controlled by invariants of symplectic geometry. For suitable symplectic structures, we obtain complete series of representations, which generalize the notion of models proposed by Gelfand.

Degenerate Whittaker models for real reductive groups
Dmitry Gourevitch

The Whittaker model is a very useful tool in the representation theory of reductive groups and in automorphic forms. However, it is known that only the "largest" representations have Whittaker models. In order to overcome this problem, for other representations various kinds of degenerate or generalized Whittaker models are considered since the 80s. Over non-archimedean fields, Moeglin and Waldspurger characterized the existence and multiplicities of these models in terms of the wave-front set. For GL(n,F) they can be also described in terms of the Bernstein-Zelevinsky derivatives. Over the archimedean fields, only partial results are known in this direction. I want to talk about these partial results, including my recent works with Siddhartha Sahi (and sometimes others) on degenerate Whittaker models, Archimedean Bernstein-Zelevinsky derivatives and the connection between them.

On the set of maximal nilpotent supports of supercuspidal representations
Yujun Qin

In this talk, we will discuss the set of maximal nilpotent supports of positive integral depth supercuspidal representations of classical groups defined over a p-adic field.

On the residual spectrum of a quasi-split group of type D4
Jing Feng Lau

In this talk, I will present the results for the residual spectrum of a quasi-split group of type D4 supported on maximal parabolics and some partial results for the residual spectrum supported on the Borel subgroup. This is work still in progress.

Abstracts (Day 3)

A quick survey on monomial ideals
Takayuki Hibi

Following the pioneering work [1] by Richard Stanley, in the late 1970s a new and exciting trend of commutative algebra, the combinatorial study of squarefree monomial ideals, broke out. Since then, it has been one of the most active areas of commutative algebra.

In my talk a quick survey on monomial ideal theory developed for the last few decades will be supplied. No special knowledge will be required to understand my talk. The systematic study on monomial ideals is achieved in [2].

[1] R. P. Stanley, The upper bound conjecture and Cohen--Macaulay rings, Stud. Appl. Math. 54 (1975), 135--142.
[2] J. Herzog and T. Hibi, ``Monomial Ideals,'' GTM 260, Springer, Berlin, 2011.

The Littlewood-Richardson triangles and highest weight vectors
Soo Teck Lee

Let $V_{n,pq} = M_{np}(\mathbb{C}) \oplus M_{nq}(\mathbb{C})$ and let $P(V_{n,pq})$ be the algebra of polynomial functions on $V_{n,pq}$. Let $GL_n(\mathbb{C}) \times GL_p(\mathbb{C}) \times GL_q(\mathbb{C})$ act on $P(V_{n,pq})$ by

$$\big((g, h_1, h_2).f\big)(X, Y) = f(g^t X h_1, g^{-1} X h_2)$$

and let $\mathcal{R}$ be the algebra generated by all $GL_n(\mathbb{C}) \times GL_p(\mathbb{C}) \times GL_q(\mathbb{C})$ highest weight vectors in $P(V_{n,pq})$. The algebra $\mathcal{R}$ is graded, and the dimension of each of its homogeneous components is equal to the multiplicity of an irreducible representation of GL_n in the tensor product of two irreducible representations of GL_n. This multiplicity is equal to the number of triangular arrays of integers of the form

$$\begin{array}{ccccccccc}
 & & & & a_{00} & & & & \\
 & & & a_{01} & & a_{11} & & & \\
 & & a_{02} & & a_{12} & & a_{22} & & \\
 & a_{03} & & a_{13} & & a_{23} & & a_{33} & \\
\vdots & & \cdots & & \cdots & & \cdots & & \ddots \\
a_{0n} & & a_{1n} & & \cdots & & \cdots & & a_{nn}
\end{array}$$

where the entries satisfy certain conditions. Such an array is called a *Littlewood-Richardson triangle*. We will construct a basis for $\mathcal{R}$ for which the elements of the basis are indexed by a set of Littlewood-Richardson triangles. This is joint work with Roger Howe and Yi Wang.

Theta correspondence between supercuspidal representations
Jiajun Ma

In this talk, I will first review J.K. Yu's construction of tamely ramified supercuspidal representations. Then I will present a conjecture on the theta correspondence of supercuspidal representations in terms of Yu's parameters. A theorem in the epipelagic case will be given at the end.

A proof of the Howe Duality Conjecture
Wee Teck Gan

We give a proof of the Howe duality conjecture in local theta correspondence for symplectic-orthogonal or unitary dual pairs in arbitrary residue characteristic. This is joint work with Shuichiro Takeda.