DEHN SURGERY ON KNOTS OF BRIDGE NUMBER THREE AND THE LENS SPACE $L(3,1)$

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Abstract. In this paper, Gordon's $L(3,1)$ conjecture is proved to be true for knots of bridge number at most three, namely, if K is a nontrivial knot in the 3-sphere with bridge number not exceeding three, then Dehn surgery on K never yields the lens space $L(3,1)$.

1. Introduction

1.1. **Main results.** Dehn surgery on a knot in a 3-manifold is an operation that first removes a tubular neighborhood of the given knot and then glues it back along the resulting torus boundary in a different way.

Dehn surgery on knots provides an effective way to construct and to study 3-manifolds. There are many important problems on this subject. During the last two decades some of them have been solved by graph theoretic analysis techniques on labeled graphs on (a pair of) surfaces in 3-manifolds. See [Go90] [Go97] [Go98] for nice surveys.

C. McA. Gordon [Go90] has conjectured that *Dehn surgery on a nontrivial knot in the 3-sphere cannot produce the lens spaces* $L(2,1), L(3,1)$ or $L(4,1)$, to which we will refer as the $L(2,1)$, $L(3,1)$, $L(4,1)$ conjecture(s).

In this paper we show that the $L(3,1)$ conjecture is true for knots of bridge number at most three, that is,

Theorem 1.1. *If K is a nontrivial knot in S^3 with bridge number not exceeding three, then Dehn surgery on K never yields the lens space $L(3,1)$.*

1.2. **Definitions about Dehn surgery on knots.** Let M be a 3-manifold, T a torus component of ∂M. The (unoriented) isotopy classes of simple closed curves on T are called *slopes* on T. The so called *Dehn filling* M along T in slope r means attaching a solid torus J ($\cong D^2 \times S^1$) along boundary to M by a homeomorphism $\partial J \longrightarrow T$ so that the image of a meridian of ∂J has slope r on T. The resulting 3-manifold is denoted $M(r)$.

For two slopes r_1, r_2 on T, their minimal geometric intersection number is called the *distance* between r_1 and r_2, denoted $\Delta(r_1, r_2)$. In particular, $r_1 = r_2$ if and only if $\Delta(r_1, r_2) = 0$.

For a knot K in M, let $M_K = \overline{M - N(K)}$, the exterior of K, where $N(K)$ is a regular neighborhood of K in M. Then $T = \partial N(K) \subset \partial M_K$ is a torus component

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of ∂M_K. For a slope r on T, $M_K(r)$ is called the *surgered manifold* after r-Dehn surgery on K. In the case $M = S^3$, we abbreviate $M_K(r)$ as $K(r)$.

For a knot K in S^3, $T = \partial M_K$ has a meridian slope μ and a standard longitude slope λ. In the μ–λ coordinates of $H_1(T)$, the slopes on T have a natural parametrization in $\mathbb{Q} \cup \{\infty\}$, i.e. $r \leftrightarrow a/b$ if and only if $[r] = a[\mu] + b[\lambda]$, where a, b are relatively prime integers; in particular, ∞ denotes $1/0$. Thus $K(\infty) \cong S^3$.

In general, for slopes a/b and c/d on T, $\Delta(a/b, c/d) = |ad - bc|$. In particular, $\Delta(r, \infty) = 1$ if and only if r is an integer.

For the trivial knot O in S^3, the surgered manifolds on O are as follows:

$$O(1/n) \cong S^3; \quad O(0) \cong S^2 \times S^1; \quad O(p/q) \cong L(p,q), \text{ where } p \geq 2.$$

For a general knot K in S^3, we have

$$H_1(K(p/q)) \cong \mathbf{Z}_p, \quad \text{where } p \geq 2.$$

1.3. **On producing surgered 3-manifolds with cyclic fundamental groups,** there is the following celebrated

Cyclic Surgery Theorem [CGLS] *Let M be an orientable, connected, compact 3-manifold with ∂M a torus. Suppose M is not a Seifert fiber space. If $\pi_1(M(r_1)), \pi_1(M(r_2))$ are cyclic for slopes r_1, r_2 on ∂M, then $\Delta(r_1, r_2) \leq 1$ (hence there are at most three such slopes).*

In the case of satellite knots in the 3-sphere there is a complete result on surgered manifolds with cyclic fundamental groups.

Theorem [BL][Wa][Wu] *Let K be a satellite knot in S^3. Then $\pi_1(K(r))$ (where $r \neq \infty$) is cyclic if and only if K is a $(2pq \pm 1, 2)$-cable over the (p,q)-torus knot, and $r = 4pq \pm 1$. In this case, $K(r) \cong L(4pq \pm 1, 4q^2)$.*

Regarding producing lens spaces by Dehn surgery on a knot in S^3, there is the following

Conjecture [Go90] *Let K be a hyperbolic knot in S^3. If $K(r)$ is a lens space, then K is a double-primitive knot defined and listed in* [Be].

This conjecture, which is still open, is much stronger than the $L(2,1), L(3,1), L(4,1)$ conjecture mentioned before.

1.4. **Strategy for proving Theorem 1.1.** The proof of Theorem 1.1 will be given in §4. It makes use of Gordon-Luecke's graph theoretic analysis for the intersection graph of two properly chosen surfaces in the exterior of the knot. Explicitly, assuming for contradiction that the surgered manifold $K(r)$ is homeomorphic to the lens space $L(3,1)$ for some slope $r \in \mathbf{Q}$. By homological reason and by the Cyclic Surgery Theorem [CGLS], we only need to consider the two slopes $r = \pm 3$. To carry out the labeled graph theory analysis, we may choose a Gabai's 2-sphere in S^3 and a 2-sphere in the surgered manifold which is the boundary of a regular neighborhood of a minimal 3-fold dunce cap in the surgered manifold. The graph theory analysis will give a so-called extended Scharlemann cycle which allows us to construct a new 3-fold dunce cap with fewer intersections with the core of the surgery in the surgered manifold, thus arriving at a contradiction. Detailed descriptions of the surfaces involved will be given in §3.

Note that there are few known results (see [Te], [GT] for some recent ones) on the $L(3,1)$ Conjecture for lack of appropriate surfaces to carry out the graph theory analysis; our consideration of 3-fold dunce cap seems to be a new idea.

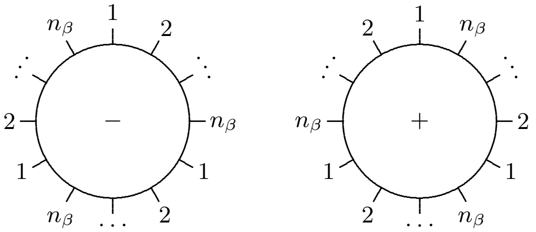

FIGURE 1. Negative and positive fat vertices in Γ_α with $\Delta = 3$

2. LABELED GRAPHS ON SURFACES IN 3-MANIFOLDS

The labeled graph method in the study of Dehn surgery on knots originated in Litherland's study of Dehn surgery on satellite knots [Lit]. Later on C. McA. Gordon and John Luecke developed the now standard techniques of graph analysis to prove part of the Cyclic Surgery Theorem [CGLS] and then the Knot Complement Conjecture [GL89].

In this section we briefly summarize the construction and basic properties of (a pair of) labeled graphs on surfaces. Standard references are [CGLS, Chapter 2], [GL89] and [Go97].

Let M be an orientable 3-manifold with $\partial M = T$, a torus. Let r_1, r_2 be two slopes on T, and $\Delta = \Delta(r_1, r_2)$. We use α, β (only in this section) to denote 1 or 2, with the convention that when they both appear, $\{\alpha, \beta\} = \{1, 2\}$.

Let $S_\alpha \subset M(r_\alpha) = M \cup J_\alpha$ be a closed (orientable) surface such that S_α intersects the solid torus J_α at a collection of n_α disjoint meridional disks. Then $P_\alpha = S_\alpha \cap M$ is a punctured surface with n_α boundary components each of which has slope r_α on T. After an isotopy we may assume that P_α and P_β intersect transversely and each component of ∂P_α intersects each component of ∂P_β at Δ points.

Number the components of ∂P_α $1, 2, \cdots, n_\alpha$ in the order that they appear on T. Then as we travel along a component of ∂P_β (anticlockwise or clockwise) the components of ∂P_α we encounter are $1, 2, \cdots, n_\alpha, \cdots\cdots, 1, 2, \cdots, n_\alpha$ where each number appears Δ times. See Figure 1.

Then the collection of all the arc components of $P_1 \cap P_2$ gives a graph Γ_α on S_α so that: (i) the vertices of Γ_α correspond to the components of ∂P_α (called *fat vertices*); and (ii) the edges of Γ_α correspond to arc components of $P_1 \cap P_2$. Note that the valency of each vertex of Γ_α is $\Delta\, n_\beta$, and that each edge of Γ_α corresponds to an edge of Γ_β (they come from the same arc of $P_1 \cap P_2$).

Two vertices of Γ_α are called *parallel* if when considered on T the corresponding two components of ∂P_α have the same orientation induced from an orientation of P_α; otherwise, they are *antiparallel*.

Since M, P_1, and P_2 are orientable, each arc of $P_1 \cap P_2$ connects intersection points of ∂P_1 and ∂P_2 of opposite signs. Thus we have the following vitally important **parity rule**:

An edge of Γ_α *connects parallel vertices in* Γ_α *if and only if the corresponding edge of* Γ_β *connects antiparallel vertices in* Γ_β.

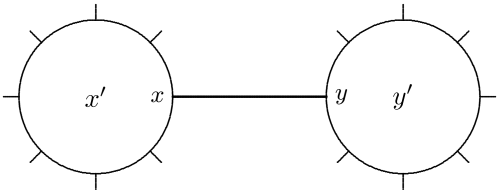

FIGURE 2. Labelling endpoints of edges

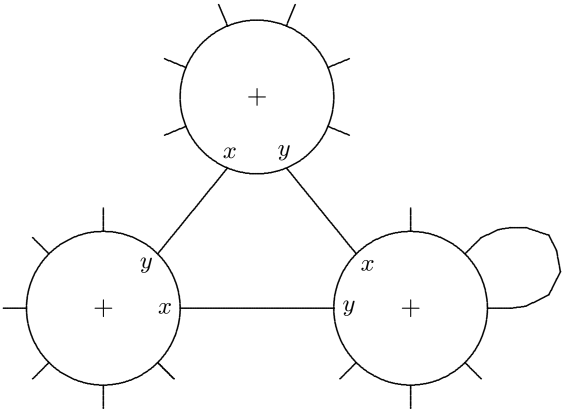

FIGURE 3. A Scharlemann cycle (of length three) and a trivial loop

For an edge e of Γ_α, we label each endpoint of e by the vertex in Γ_β which corresponds to that endpoint of e. Thus each endpoint of each edge of Γ_α is labelled by the vertex in Γ_β corresponding to that endpoint.

A cycle in Γ_α is a subgraph of Γ_α which is homeomorphic to a circle. Let x be a vertex of Γ_β. A cycle σ of Γ_α is called an *x-cycle* if all the vertices of σ are parallel, and for some orientation of σ, each edge of σ has label x at its tail.

An x-cycle σ of Γ_α is called a *Scharlemann cycle* if σ bounds a disk D in Γ_α such that $(\text{int}D) \cap \Gamma_\alpha = \emptyset$, i.e. in the interior of D there are neither vertices nor edges of Γ_α. The number of edges in a Scharlemann cycle is called its *length.* In particular, a *trivial loop* is a Scharlemann cycle of length one.

It follows that if a Scharlemann cycle of Γ_α has label x at the tail of each of its edges, then each edge has label y at its head where x, y are adjacent and antiparallel vertices of Γ_β. See Figure 3. The labels x, y are called the *label pair* of the Scharlemann cycle, and the Scharlemann cycle is said to be an x-Scharlemann cycle or an $\{x, y\}$-Scharlemann cycle.

Two edges e, e' of Γ_α are called *parallel* if the corresponding two arcs in $P_1 \cap P_2$ together with two arcs in ∂P_α cobound a disk F in P_α. Note that F may contain other edges but no vertices of Γ_α. See Figure 4.

An x-cycle in Γ_α is called a *great x-cycle* if it bounds a closed disk in S_α such that all vertices of Γ_α in it are parallel.

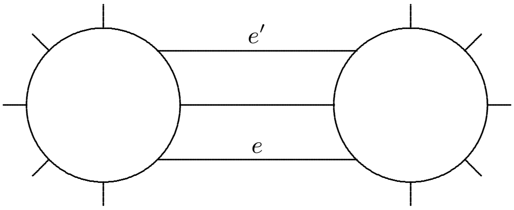

FIGURE 4. Parallel edges

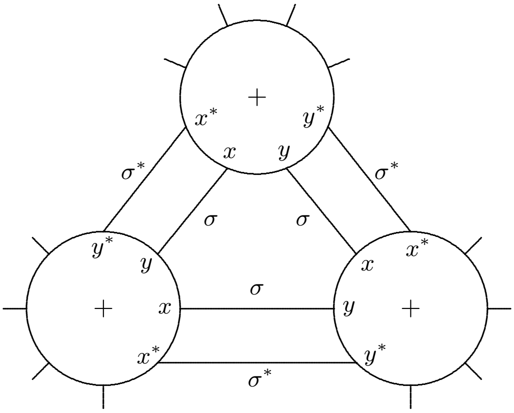

FIGURE 5. An extended Scharlemann cycle σ^*

Lemma 2.1. [CGLS, Lemma 2.6.2] *If* Γ_α *contains a great* x*-cycle, then it contains a Scharlemann cycle.*

It should be noted that the Scharlemann cycle in the above lemma need not be an x-Scharlemann cycle. The presence of a Scharlemann cycle has the following topological consequence.

Lemma 2.2. [CGLS, Lemma 2.5.2] *If* Γ_α *contains a Scharlemann cycle* σ *of length greater than one, and the edges in* Γ_β *which correspond to the edges of* σ *are contained in a disk in* Γ_β*, then* $M(r_\beta)$ *has a lens space summand.*

A cycle σ^* in Γ_α is called an *extended Scharlemann cycle* if the edges of σ^* are parallel and adjacent to the edges of a Scharlemann cycle. See Figure 5.

3. FINDING APPROPRIATE SURFACES

Usually the graph analysis gives rise to Scharlemann cycles or similar properties. Since trivial Scharlemann cycles (i.e. trivial loops) has no useful topological consequence, we need to find appropriate surfaces so that the pair of graphs constructed as in §2 on these surfaces contain no trivial loops.

(A) The surface in $K(\infty) = S^3$ is a 2-sphere (meeting K transversely) coming from Gabai's notion of *thin position* of K in S^3, which we describe as follows.

Fix two points of S^3 and a height function $h : S^3\backslash\{\text{the two points}\} \cong \mathrm{S}^2\times\mathbb{R} \to \mathbb{R}$. Let S_t denote the level 2-sphere at height t, i.e. $S_t = S^2 \times t$. Isotope K so that K is in a Morse position with respect to h, i.e., K misses the two points and the height function restricted to K has only non-degenerate critical points. Given a Morse presentation of K, let $S_{t_1}, S_{t_2}, \cdots, S_{t_n}$ be level 2-spheres, one between each pair of adjacent critical levels. Define the *width*, $w(K)$, of the knot K to be the minimal value of $\sum_{i=1}^{n} |S_{t_i} \cap K|$ over all Morse presentations of K. A *thin presentation* of K is then a Morse presentation of K which realizes the width of K.

Gabai [Ga, Lemma 4.4] shows that a thin presentation of K allows one to find a level 2-sphere $\hat{Q} \subset S^3$ so that $Q = \hat{Q} \cap M_K$ meets a given surface $P \subset M_K$ transversely and *essentially* (namely, there is no trivial loops in the previously constructed graph G_P on $\hat{P}$).

Proposition 3.1. [Ga, Lemma 4.4] *Let $P \subset M_K$ be a properly embedded surface such that each component of ∂P has slope r on $T = \partial M(K)$. Then there is a level 2-sphere $\hat{Q}$ in a thin presentation of K such that*

1) *each component of ∂Q, where $Q = \hat{Q} \cap M_K$, has slope ∞ on T;*

2) *P and Q intersect transversely;*

3) *each component of ∂P meets each component of ∂Q transversely in Δ points where $\Delta = \Delta(r, \infty)$;*

4) *no arc of $P \cap Q$ is boundary parallel on P.*

(B) Surfaces in the surgered manifold $K(r)$.

An *n-fold dunce cap* (see [Mu] page 41 for instance) is topologically the unit disk (or a regular n-gon) with its boundary folded by a $2\pi/n$-rotation; it consists of one 0-cell, one 1-cell and one 2-cell. We call the 1-skeleton of this cell decomposition the *spine* of the dunce cap.

Since $K(r)$ is homeomorphic to the lens space $L(3, 1)$, there are embedded 3-fold dunce caps in $K(r)$. Consider an embedded 3-fold dunce cap Ξ in $K(r)$ which meets K' transversely in its open 2-cell. Note that K' must meet Ξ since M_K contains no 3-fold dunce caps. Choose Ξ so that $|K' \cap \Xi|$ is minimal among all such 3-fold dunce caps in $K(r)$.

Let $\hat{P}$ be the boundary of a regular neighborhood of Ξ in $K(r)$. Then $\hat{P}$ is a 2-sphere. We may assume that K' meets $\hat{P}$ transversely by choosing the regular neighborhood properly.

Lemma 3.2. *The graph G_Q on $\hat{Q}$ contains no trivial loops.*

Proof Suppose G_Q contains a trivial loop. Then there is an arc α in $P\cap Q$ and an arc β in ∂Q such that $\alpha\cup\beta$ bounds a disk D on Q. Note that α is parallel on Q to an arc α' in $Q\cap(\Xi\cap M_K)$ since $\hat{P}$ is the boundary of a regular neighborhood of Ξ. If α' is contained in the open 2-cell of Ξ, then pushing (the part α of) K' along D reduces $|K' \cap \Xi|$ by 2 (see Figure 6), contradicting the minimality of $|K' \cap \Xi|$. Otherwise, α' crosses the spine (i.e. the 1-skeleton) of Ξ once (see Figure 6). Now pushing K' along D across the spine of Ξ reduces $|K' \cap \Xi|$ by 1, again a contradiction. This proves Lemma 3.2. □

4. Proof of Theorem 1.1

In this section we prove Theorem 1.1 which states that $L(3, 1)$ Conjecture is true for knots with bridge number up to three.

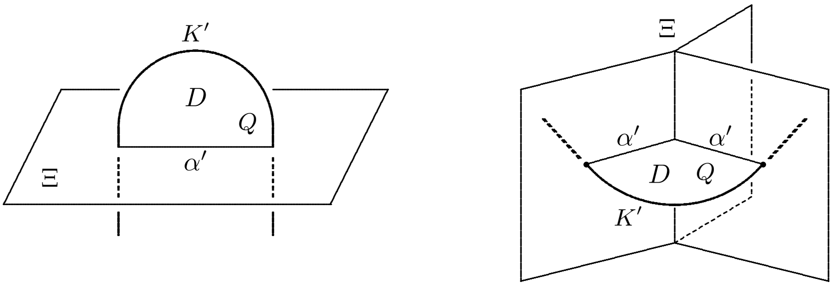

FIGURE 6. Pushing K' along D

Proof of Theorem 1.1. Suppose $K(r) \cong L(3,1)$ for some slope r. Then K is not a torus knot by [Mo], hence M_K is not Seifert fibered. Thus $\Delta = \Delta(\infty, r) = 1$ by the Cyclic Surgery Theorem [CGLS].

Choose an embedded 3-fold dunce cap Ξ in $K(r)$ as in Section 3 and let $\hat{P}$ be the boundary of a regular neighborhood of Ξ in $K(r)$. Let $P = \hat{P} \cap M_K$.

Then, by Proposition 3.1, there is a level 2-sphere $\hat{Q}$ in a thin presentation of K such that each arc in $P \cap Q$ (where $Q = \hat{Q} \cap M_K$) is essential in P. On the other hand, each arc in $P \cap Q$ is essential in Q by Lemma 3.2.

Let p (resp. q) be the number of components of ∂P (resp. ∂Q). Then p and q are even since $\hat{P}$ and $\hat{Q}$ are separating in $K(r)$ and S^3 respectively.

Note that $p \geq 4$ since if $p = 2$ then K is a torus knot or a cable knot which is impossible by [Mo] and [Go83].

Claim 1 $q \leq 6$.

This is because if $q > 6$, then a bridge three presentation of K would be thinner than a thin presentation, a contradiction.

Now consider the pair of graphs G_P, G_Q constructed as in §3. Here $\Delta = \Delta(\infty, r) = 1$.

By [GL89, Proposition 2.8.1], G_Q contains a Scharlemann cycle, say σ. Then applying the same proposition to the graph G_Q with the disk face bounded by σ pierced, we obtain another Scharlemann cycle, say σ'. Since $K(r) \cong L(3,1)$ these two Scharlemann cycles must have length three and the same label pair. In fact we must have $q = 6$ since in the case $q \leq 4$ the Scharlemann cycles would have length two which is impossible by Lemma 2.2.

Without loss of generality, let the label pair be $1, 2$ where $1, 2$ are two adjacent vertices of G_P. Then the six edges in G_P corresponding to the six edges of σ, σ' connect the two vertices $1, 2$ of G_P and divide G_P into six disks; see Figure 7.

Since $p \geq 4$, the interior of one of these disks, say D, contains some vertices of G_P. Now consider the graphs of the intersection of $\mathrm{int} D$ and Q. The graph on $\mathrm{int} D$ is G_P restricted to $\mathrm{int} D$, denoted G_D. The graph on $\hat{Q}$ is $G_Q(L)$ where L is the set of vertices of G_P in $\mathrm{int} D$ and $G_Q(L)$ is the subgraph of G_Q consisting of all (fat) vertices of G_Q and the edges of G_Q with end labels in L.

Then Gordon and Luecke's graph analysis [GL89, Proposition 2.8.1] applies to this pair of graphs and implies that $G_Q(L)$ contains a Scharlemann cycle σ^* of

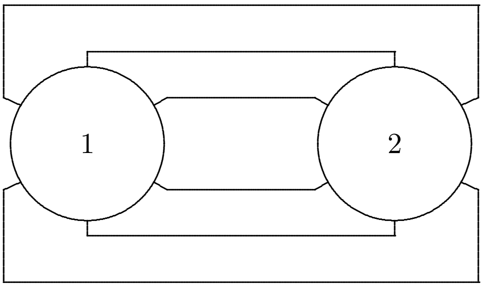

FIGURE 7. The six edges

length three with label pair x^*, y^* in L. Note that the disk face F of $G_Q(L)$ bounded by σ^* is not a face of G_Q (otherwise G_Q would contain two Scharlemann cycles on distinct label pairs $\{1,2\}$ and $\{x^*, y^*\}$, which is impossible by [GL96]).

Hence the face F of $G_Q(L)$ contains a Scharlemann cycle of G_Q, say σ. Thus we have found an extended Scharlemann cycle, σ^*, in G_Q.

In the rest of the proof we show that the existence of an extended Scharlemann cycle in G_Q contradicts the minimality of the 3-fold dunce cap Ξ in $K(r)$.

Let σ^* be an extended Scharlemann cycle in G_Q, with edges e_1^*, e_2^*, e_3^*. Then the label pair of σ^* must be $p, 3$. Let the Scharlemann cycle in the interior of σ^* be σ (with label pair $1, 2$). The three edges in G_P which correspond to the edges of the Scharlemann cycle σ divide the graph G_P into three disks, D_1, D_2, D_3.

Claim 2 *For each $i = 1, 2, 3$, the interior of D_i contains at most $p/2 - 1$ vertices of G_P.*

To prove Claim 2, let us suppose for contradiction that D_3, say, contains more than $p/2 - 1$ vertices of G_P. Then

$$|K' \cap (D_1 \cup D_2)| < p/2 - 1.$$

Let f be the disk face of G_Q which the Scharlemann cycle σ bounds. Then

$$\partial f = e_1 \cup e_1' \cup e_2 \cup e_2' \cup e_3 \cup e_3',$$

where e_1, e_2, e_3 are the edges of σ and e_1', e_2', e_3' are arcs in ∂Q. See Figure 8.

Let A_{12} be the annulus on $T = \partial M_K$ between the two components of ∂P numbered 1 and 2 (namely, those correspond to the fat vertices $1, 2$ of G_P). Then the arcs e_1', e_2', e_3' lie in A_{12} and they divide A_{12} into three disks D_1', D_2', D_3'. Here the indices are chosen so that D_1 lies between e_2, e_3, and D_1' lies between e_2', e_3', etc. See Figure 9.

Let f_2 be the disk in $K(r)$ which corresponds to the fat vertex 2. Then

$$\Xi' = f \cup D_1 \cup D_2 \cup D_3' \cup D_1' \cup f_2$$

is an embedded 3-fold dunce cap in $K(r)$. The spine of Ξ' is the union of the arcs e_2', e_3 and the arc $D_1 \cap f_2$. See Figure 9.

Note that K' meets Ξ' transversely in its open 2-cell. Now we have

$$|K' \cap \Xi'| < (p/2 - 1) + 1 = p/2 = |K' \cap \Xi|,$$

contradicting the minimality of Ξ. This proves Claim 2.

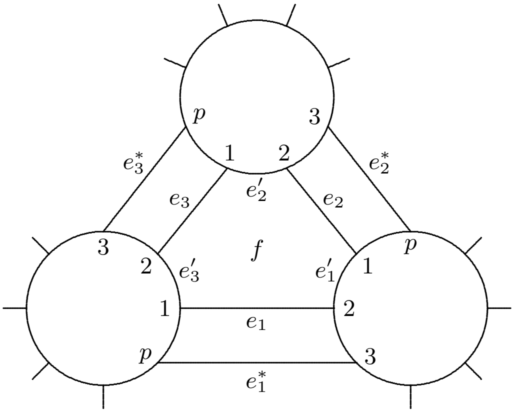

Figure 8. The extended Scharlemann cycle σ^* with edges e_1^*, e_2^*, e_3^*

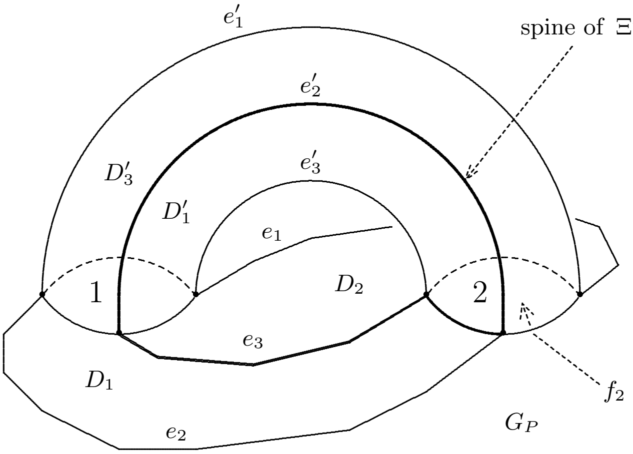

Figure 9. Construction of the new dunce cap Ξ'

Since the two vertices $p, 3$ of G_P are connected by the edges e_1^*, e_2^*, e_3^* in G_P, they all lie in the same open disk, say D_3. By Claim 2,

$$|K' \cap D_3| \leq p/2 - 1.$$

Now the three edges e_1^*, e_2^*, e_3^* in G_P divide G_P into three disks: D_1^*, D_2^*, D_3^*. The interior of one of them, say D_3^*, contains the vertices $1, 2$ of G_P. Then D_1^*, D_2^* are both contained in the interior of D_3. Hence

$$|K' \cap (D_1^* \cup D_2^*)| \leq (p/2 - 1) - 2,$$

since at least the two vertices $p, 3$ are further excluded.

Let A_{p3} be the annulus on $T = \partial M_K$ between the two components of ∂P numbered p and 3 (namely, those correspond to the fat vertices $p, 3$ of G_P) such that A_{p3} contains the two components of ∂P numbered 1 and 2 (namely, those correspond to the fat vertices $1, 2$ of G_P). Then a similar construction as in the proof of Claim 2 gives an embedded 3-fold dunce cap Ξ^* in $K(r)$ such that K' meets Ξ^* transversely in its open 2-cell and that

$$|K' \cap \Xi^*| \leq (p/2 - 1) - 2 + 1 < p/2 = |K' \cap \Xi|,$$

contradicting the minimality of Ξ. Theorem 1.1 is therefore proved. □

5. Some remarks

The $L(2,1)$ Conjecture (also called the $\mathbb{R}P^3$ Conjecture) was proved true for knots with bridge number not exceeding five by J. Hoffman [Ho]. In this case suppose $K(r) \cong L(2,1)$. Note that $K(r)$ contains a projective plane. The surface $\hat{P}$ is chosen as the boundary of a regular neighborhood of a minimal projective plane in $K(r)$; and $\hat{Q}$ is a level 2-sphere in a thin presentation of K. Then J. Hoffman (see [Ho],[Ho98]) was able to show that the graph G_Q cannot contain a strict great x-cycle (i.e. a great x-cycle which is not a Scharlemann cycle). This is a key ingredient in his proof of the $L(2,1)$ conjecture for knots with bridge number not exceeding five. In our case of $K(r) \cong L(3,1)$, however, we have no such an exclusion result; in fact, it seems possible for G_Q to have strict great x-cycles. This is the main reason why the method in this paper fails to prove the $L(3,1)$ Conjecture for knots with bridge number greater than three.

On the other hand, we have noticed the recent achievement of P. Kronheimer, T. Mrowka, P. Ozsváth, and Z. Szabó [KMOS] that Gordon's $L(2,1)$, $L(3,1)$, $L(4,1)$ conjectures among others have been proved to be true for all knots in the 3-sphere. Indeed, combining with the results of [GT], they have obtained the following

Theorem ([KMOS, Corollary 8.4]) *If K is a nontrivial knot in S^3 and is not the trefoil knot, and $r \in \mathbf{Z}$ is an integral slope with $|r| < 9$, then the surgered manifold $K(r)$ is not a lens space.*

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