GEK1518 Mathematics in Art and Architecture, 2003/04 II, Tutorial 8 Solutions

1. The shape of “An” paper is determined by the following rules.
   - “An” is a rectangle.
   - If you fold “An” along its longest side you get “A(n+1)”.
   - “An” is similar to “A(n+1)”.
   - The area of A0 is 1m²

What is the size of A4 paper?

**Solution:** Suppose A0 paper has width \( w \) and height \( h \). Let us write that as

\[
A0: (w, h).
\]

Since A1 is obtained by folding A0 height-wise, it follows that

\[
A1: (h/2, w).
\]

But since the two are similar, we must have

\[
\begin{align*}
w/ h &= h/2, \\
2w^2 &= h^2, \\
2 &= (h/w)^2, \\
h/w &= \sqrt{2}.
\end{align*}
\]

It follows that all the “An” shapes have a height/width ratio of \( \sqrt{2} \).

Since A0 has area 1m², we have

\[
\begin{align*}
hw &= 1, \\
\sqrt{2}w^2 &= 1, \\
w^2 &= 2^{-1/2}, \\
w &= 2^{-1/4}.
\end{align*}
\]

It then follows that \( h = 2^{1/4} \), so

\[
A0: (2^{-1/4}, 2^{1/4}).
\]

By folding, we see that
A1: \((2^{-3/4}, 2^{-1/4})\),
A2: \((2^{-5/4}, 2^{-3/4})\),
A3: \((2^{-7/4}, 2^{-5/4})\),
A4: \((2^{-9/4}, 2^{-7/4}) = (0.210, 0.297)\).

As a check, you see that the area of A4 is \(2^{-4} = 1/16\), which is correct since we have divided A0 four times.

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