MA3264   Homework 1
(Due date: 6:00pm, September 6, 2013 (Friday))

1. You wish to buy a new car upon graduation from college. You narrow your choices to a Saturn, Cavalier, and Tercel. Each company offers you its prime deal:

<table>
<thead>
<tr>
<th></th>
<th>Price</th>
<th>Down</th>
<th>Interest Rate</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Saturn</td>
<td>$11,990</td>
<td>$1,000</td>
<td>3.5% yearly</td>
<td>60 months</td>
</tr>
<tr>
<td>Cavalier</td>
<td>$11,550</td>
<td>$1,500</td>
<td>4.5% yearly</td>
<td>60 months</td>
</tr>
<tr>
<td>Tercel</td>
<td>$10,900</td>
<td>$500</td>
<td>6.5% yearly</td>
<td>48 months</td>
</tr>
</tbody>
</table>

Establish a mathematical model to determine which car to buy and how much you pay each month and in total to buy the car for the following cases:

(i) You are able to spend at most $475 a month on a car payment.
(ii) You want to pay the minimum cost for the car in total with its prime deal.

2. Sociologists recognize a phenomenon called social diffusion, which is the spreading of a piece of information, a technological innovation, or a cultural fad among a population. The members of the population can be divided into two classes: those who have the information and those who do not. In a fixed population whose size is known, it is reasonable to assume that the rate of diffusion is proportional to the number who have the information times the number yet to receive it. Establish a model for the dynamics of the number of people who have the information. Find the time when half of the people have the information.

3. Suppose that the fish population $P(t)$ in a lake is attacked by a disease at time $t = 0$, with the result that the fish cease to reproduce (so that the birth rate is $\beta = 0$) and the death rate $\delta$ (deaths per week per fish) is thereafter proportional to $1/\sqrt{P}$. Formulate a model for the dynamics of the population of the fish in the lake.

(i) Find the time when the population of the fish is at the half of the initial population.
(ii) If there are initially 900 fish in the lake and 441 were left after 6 weeks, how long did it take all the fish in the lake to die?

4. Purchasing Various Trucks – A truck company allocated $800,000 for the purchase of new vehicles and is considering three types: Vehicle A has a 10-ton payload capacity and is expected to average 45 mph; it costs $26,000. Vehicle B has a 20-ton payload capacity and is expected to average 40 mph; it costs $36,000. Vehicle C is a modified form of Vehicle B and carries sleeping quarters for one driver. This modification reduces the capacity to an 18-ton payload and raises the cost to $42,000, but it operating speed is still expected to average 40 mph.

Vehicle A requires a crew of one driver and, if driven on three shifts per day, could be operated for an average of 18 hr per day. Vehicles B and C must have crews of two drivers each to meet local legal requirements. Vehicle B could be driven an average of 18 hr per day with three shifts, and Vehicle C could be driven 21 hr per day with three shifts. The company has 150 drivers available each day to make up crews and will not be able to hire additional trained crews in the near future. The local labor union prohibits any driver from working more than one shift per day. Also, maintenance facilities are such that the total
number of vehicles must not exceed 30. Establish a mathematical model to help determine the number of each type vehicle the company should purchase to maximize its shipping capacity in ton-miles per day.

5. John Smith is responsible for periodically buying new trucks to replace older trucks in his company’s fleet of vehicles. He is expected to determine the length of time a truck should be retained to minimize the average cost of owning the truck. Assume the purchase price of a new truck is $9,000 with trade-in. Also assume the maintenance cost (in dollars) per truck for \( t \) years can be expressed analytically by the following empirical model:

\[
C(t) = 640 + 180(t + 1)t, \quad t \geq 0,
\]

where \( t \) is the time in years the company owns the truck.

(a) Determine \( E(t) \), the total cost function for a single truck retained for a period of \( t \) years.

(b) Determine \( E_A(t) \), the average annual cost function for a single truck kept in the fleet for \( t \) years.

(c) Graphically depict \( E_A(t) \) as a function of \( t \). Justify the shape of your graph.

(d) Analytically determine \( t^* \), the optimal period a truck should be retained in the fleet. Remember that the objective is to minimize the average cost of owning a truck.

(e) Suppose we have to round \( t^* \) to the nearest whole year. In general, would it be better to round up or round down? Justify your answer.

6. A local daily newspaper has recently been acquired by a large media conglomerate. The paper currently sells for $1.50/week and has a circulation of 80,000 subscribers. Advertising sells for $250/page, and the paper currently sells 350 pages/week (50 pages/day).

(a) The new management is looking for ways to increase profits. It is estimated that an increase of 10 cents/week in the subscription price will cause a drop in circulation of 5,000 subscribers. Increasing the price of advertising by $100/page will cause the paper to lose approximately 50 pages of advertising per week. The loss of advertising will also affect circulation, since one of the reasons people buy the paper is for the advertisements. It is estimated that a loss of 50 pages of advertisements per week will reduce circulation by 1,000 subscriptions. Find the weekly subscription price and the advertising price that will maximize profit.

(b) Advertisers who currently place advertisements in the newspaper have the option of using direct mail to reach their customers. Direct mail would cost the equivalent of $500/page of newspaper advertising. How does this information alter your conclusion in part (a)? Because of this, management has decided not to increase the price of advertising beyond $400/page. Find the weekly subscription price and the advertising price that will maximize profit again.

(c) Now we look at the newspaper’s business expenses. The current weekly expenses for the paper are $80,000 for the editorial department (news, features, editorials), $30,000 for the sales department (advertising), $30,000 for the circulation department, and $60,000 in fixed costs (mortgage, utilities, maintenance). The new management is considering cuts in the editorial department. It is estimated that the paper can operate with a minimum of a $40,000/week editorial budget. Reduce the editorial budget will save money, but it will also
affect the quality of the paper. Experience in other markets suggests that the paper will lose 2% of its subscribers and 1% of its advertisers for every 10% cut in the editorial budget. Management is also considering an increase in the sales budget. Recently the management of another paper in a similar market expanded its advertising sales budget by 20%. The result was a 15% increase in advertisements. The sales budget may be increased to as much as $50,000/week, but the overall budget for business expenses will not be increased beyond the current level of $200,000/week. Find the editorial and sales budget that maximize profit. Assume that the subscription price remain at $1.50/week, and the advertising price stays at $250/page.