How do you test your code?

If you do not know what the solutions should look like, then how do you test your program?

1) Make sure that you obtain the convergence that you expect. This can eliminate some basic programming errors.

To check convergence you can e.g. study the solution in one point as the grid is refined and look how the difference \( |u_{ij}^{h/2} - u_{ij}^h| \) behaves as you increase the number of points.

2) Construct a problem to which you know the exact solution.

Ex. \( u_{xx} + u_{yy} = f(x,y), \quad u(x,y) = 0 \) on \( \partial \Omega \)

Let the exact solution be a function that fulfills the boundary conditions, \( u(x,y) = \sin(\pi x)\sin(\pi y) \)

Given \( u(x,y) \) compute \( f(x,y) \) such that the equations fulfills the exact solution. In this case \( f(x,y) = -2\pi^2 \sin(\pi x)\sin(\pi y) \).

Solve the equation numerically with \( f(x,y) \) as above and compare the numerical solution to the exact solution.