Lecture 18

Summary of the course

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Fourier series/Discrete Fourier transform

- How are the coefficients computed
- What is the orthogonal condition
- Different notions of convergence
- What is Gibbs phenomenon
- How is the decay of the Fourier coefficients related to the regularity of the expanded function
- What is the discrete Fourier transform, DFT
- How does the coefficients in the DFT relate to the Fourier coefficient
- What is aliasing error
- Computational cost of evaluating the DFT directly
- Basic ideas behind FFT and computational cost
Polynomial transform in (-1,1) and unbounded domains

- Polynomial transform of a function
  - Chebyshev
  - Legendre
  - Hermite
  - Laguerre
  (No need to know recursion formulas/formulas for different polynomials by heart)
- Compute the coefficients in the transform
- Discrete polynomial transform
- Compute the coefficient in the discrete polynomial transform
- Idea behind Gauss-quadrature (no need to know formulas for discrete collocation points and weights by heart)
- Multi-dimensions

Spectral interpolation and differentiation

- Describe the idea of spectral interpolation (Fourier, Chebyshev, Legendre, Hermite, Laguerre)
- What kind of convergence is expected and how does it depend on the function that you want to interpolate
- Filtering techniques when interpolating function with sharp gradients
- Describe the idea of spectral differentiation (no need to know elements in spectral differentiation matrices by heart)
- What kind of convergence is expected and how does it depend on the function that you want to differentiate
Spectral methods for differential equations

- Describe all the steps in designing a spectral collocation or spectral Galerkin method for solving a differential equation
  - Periodic problems
  - Bounded domain
  - Unbounded domain
- Modal and nodal discretization
- Discuss the errors of such a method
- How are non-linear terms treated?
- Cost of full evaluation
- Pseudo-spectral method and aliasing errors
- How can aliasing error be removed or reduced
- Advantages and disadvantages with different methods and when to use what method

Solution techniques

- Structure of discretized operators (full, sparse, symmetric) when using different methods
- Eigenvalues of operators and their relation to performance of methods
- Iterative methods
- Pre-conditioners
- Matrix decomposition method
- Time stepping methods – stability
- Computational cost of different algorithms