Aims
The aim of this module is to present and help you understand the essentials of spectral methods and related computer algorithms and to show you how spectral methods can be used in different applications and settings. The emphasis will be on both mathematical theory as well as numerical algorithms and their efficient implementation.

Objectives
After this module you will be able to

- Formulate a spectral method/approximation of given problem (PDE)
- Identify what kind of spectral method to use for the problem at hand
- Present the advantages and limitations of different spectral approximations
- Construct efficient implementations of spectral methods in a computer program/code to solve a given problem
- Run, interpret and present the results from your spectral method computer code

Course responsible
Katarina Gustavsson

Email: matgkv@nus.edu.sg

Office S17, 0517

Office hours: Wednesday 3:00pm-4.30pm

Course literature
*Spectral and High-Order Methods with Applications*, Jie Shen and Tao Tang, Science Press, China, 2006

*Spectral Methods in Matlab*, Lloyd N. Trefethen, SIAM, USA, 2000

*Spectral Methods, Fundamentals in single domains*, C Canuto et al., Springer-Verlag, Berlin, 2006 (Can be found as electronic resource at NUS central library.)

*Spectral Methods, Evolution to complex geometries and applications to fluid dynamics*, C Canuto et al., Springer-Verlag, Berlin, 2007 (Can be found as electronic resource at NUS central library.)
Course requirements

The course consists of the following mandatory assignments

- Homeworks (2 in total)
- Project (to be done in groups of two)
- Written examination

To pass the course, you have to pass the written examination, the homeworks and the project. The final grade of the course will be based on the sum of the credits on the homework (max 30) and the exam (max 30), i.e. a total max of 60. The grade of the project will be pass or fail.

The solutions to the homeworks and the project should be handed in as written reports. The homework solutions will also be discussed in class and an oral presentation of the project is mandatory.

Please note that late hand in of homeworks will lead to a reduction in credits for the homework.

Schedule

Lectures will be on Mondays and Thursdays 2.00pm-16.00pm in S17, 0512.

Preliminary course outline

- Week 1, 08/08-12/08, Introduction
  - Introduction and some basic ideas of spectral methods
- Week 2, 15/08-19/08, Introduction
  - The Fourier system and Fast Fourier Transforms
  - Polynomial interpolation
- Week 3, 22/08-26/08, Spectral-Collocation methods
  - Differentiation matrices for Fourier collocation methods
  - Differentiation matrices for polynomial basis functions
- Week 4, 29/08-02/09, Spectral-Collocation methods
  - Fourier collocation method for PDEs
  - Review on time discretization methods
- Week 5, 05/09-09/09, Spectral-Collocation methods
  - Chebyshev collocation methods
  - Collocation methods in weak form
- Week 6, 12/09-16/09, Spectral-Galerkin methods
  Homework 1 due 12/09 2.00pm
  - Homework seminar
  - Introduction and general set up
  - Fourier spectral and pseudo-spectral method
- Week 7, 26/09-30/09, Spectral-Galerkin methods
• Legendre-Galerkin method
• Chebyshev-Galerkin method

• Week 8, 03/10-07/10, Spectral-Galerkin methods
  – E-learning week
  – Iterative methods and pre-conditioning
  – Spectral-Galerkin for higher order equations
  – Error estimates

• Week 9, 10/10-14/10, Spectral methods in unbounded domains
  – Hermite spectral methods
  – Laguerre spectral methods
  – Spectral methods with rational functions

• Week 10, 17/10-21/10, Spectral methods in multi-dimensional domains
  – Spectral-collocation in rectangular domains
  – Spectral-Galerkin in rectangular domains
  – Fast Poisson solver

• Week 11, 24/10-28/10, Applications
  Homework 2 due 27/10 2.00pm
  – Spectral methods for wave equation
  – Spectral approximation of Stokes equations
  – Spectral approximation of Navier-Stokes equations
  – Homework seminar

• Week 12, 31/10-04/11, Project

• Week 13, 07/11-11/11, Project, cont.
  Project due 10/11 2.00pm
  – Project presentations