(1) Express the following numbers in rectangular coordinates.
   (a) $\frac{-1+3i}{\sqrt{2} + i}$;
   (b) $\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)^4$;
   (c) $(-i)^{1/9}$;
   (d) $\left([-1 + i\sqrt{3}]^{1/3}\right)^3$;
   (e) $\left([-1 + i\sqrt{3}]^{1/3}\right)^{1/3}$.

(2) Use the fact that $[1, n\theta] = [1, \theta]^n$ to obtain an identity for $\cos n\theta$ in terms of $\cos \theta$ and $\sin \theta$.

(3) Find a formula for $(a + ib)^{1/2}$ by solving for $c$ and $d$ in the equation $(c + id)^2 = (a + ib)$.

(4) Graph the solution sets of the given relations.
   (a) $|\frac{z-1}{z+1}| \leq 1$;
   (b) $|z^2 - 1| = 1$. [Use polar coordinates.]

(5) For any complex numbers $u$ and $v$, show that
   $|u + v|^2 + |u - v|^2 = 2(|u|^2 + |v|^2)$.

(6) (a) Let $u$ and $v$ be complex numbers. Show that
   $|u + v|^2 = |u|^2 + 2 \text{Re}(uv) + |v|^2$.
   (b) If $|z_1| = |z_2| = |z_3| = 1$ and $z_1 + z_2 + z_3 = 0$, show that $z_1, z_2$, and $z_3$ are the vertices of an equilateral triangle in the complex plane.
   [Part (a) can be used to prove part (b)!]

(7) (a) Let $u$ and $v$ be nonzero complex numbers. Find all possible values of $\text{Arg} \frac{u}{v} - (\text{Arg} u - \text{Arg} v)$.
   [Give examples to show that all your answers are indeed possible.]
   (b) Suppose that $|z| = 1$. Show that
   $$\text{Arg} \left(\frac{z - 1}{z + 1}\right) = \begin{cases} \frac{\pi}{2} & \text{if } \text{Im } z > 0 \\ -\frac{\pi}{2} & \text{if } \text{Im } z < 0. \end{cases}$$
   [Part (b) takes a little bit of plane geometry.]