In 1937, nearly 75 years after Boole’s death, Claude Shanon, a student at MIT, recognized [this] connection between electronic circuits and Boolean Algebra. He transferred the two logic states to electronic circuits by assigning different voltage levels to each state. This connection was essential for the design of digital computers.
Circuit Diagrams:

\[ A = B \land C \]
\[ D = E + F \]

\[ D = E \lor F \]
\[ G = \overline{H} \]

\[ G = \neg H \]
\[ E = (A \land B) \lor (C \land D) \]
\[ J = (F \lor G) \land (H \lor D = I) \]
\[ D = A + (\overline{A} + B) + (A + C) \]
With its application in light switches and computers, Boolean Algebra has become a much more important part of mathematics. Circuits, the fundamental building blocks of computers, as well as sets and combinations, are dependent upon Boolean Algebra and have caused its usefulness to rise.

One must note with fascination (again) that a seemingly mundane math. could be an integral part of what is regarded as perhaps the most significant technology of the 20th century. It makes one wonder and speculate what other aspects of math. will be brought back to life to serve some useful purpose.
Some interesting articles:

(1) The Calculus of Logic, available at

http://www.maths.tcd.ie/pub/HistMath/People/Boole/CalcLogic/CalcLogic.pdf

(2) Boole meet Gates, A Look at the History of Computers, and the Role of Boolean Algebra

http://personal.nbnet.nb.ca/michaels/boole.htm
Major Facilities for Mathematical Thinking and Understanding.

(5) Stimulus-response.

Common examples can be quoted:

\[ 18 \times 19 = ? \]
\[ 531 \div 3 = ? \]

\[(x + 1)^2 = x^2 + 2x + 1, \]
\[(x + 1)^4 = x^4 + 4x^3 + 6x^2 + 4x + 1. \]
We would like to see how these simple considerations can lead to interesting mathematics.
We say that an integer $p > 1$ is a *prime number*, or a *prime*, if $p$ is not divisible by any positive integer $d$ satisfying $1 < d < p$. e.g. $7, 11, 23, 29,\ldots$

The story of primes is a long one. For example, the largest prime (so far) is discovered in 2003:

$$2^{20996011} - 1$$

It has more than 6 million digits! (A Mersenne prime $2^m - 1$.)
As of 2004, the state of the art in primality testing can certify a 4769-digit prime in approximately 2000 hours of computation (or nearly three months of uninterrupted computation) on a 1 GHz processor using this technique.

A handful of the ~ 6 million digits in the newly discovered prime number. It would need a book thicker than the Singapore phone directory to print all the numerals.
In Book IX of the *Elements*, Euclid *proves* that there are infinitely many prime numbers.

Find all the (integer) factors of 2491.

\[= 47 \times 53 \text{ (NP-problem).}\]

We have the following result which is intuitive enough:

*For any positive integer \( n > 1 \), \( n \) can be expressed as a product of primes. The expression is unique apart from rearrangements.*

\[n = p_1^{\alpha_1} \cdot p_2^{\alpha_2} \cdots p_r^{\alpha_r}\]

Here \( p_1, p_2, \cdots, p_r \) are distinct primes,

and \( \alpha_1, \alpha_2, \cdots, \alpha_r \) are positive integers.
Then we have the Goldbach Conjecture (1742), which says that any even integer bigger than 3 can be written as the sum of two primes.

e.g. \(4 = 2 + 2\);
\(6 = 3 + 3\);
\(8 = 3 + 5\);
\(10 = ? + ?\);
\(100 = ? + ?\).

\(100 = 47 + 53 = 17 + 83\).
Faber and Faber offered $1 million to anyone who proved Goldbach’s conjecture between March 20, 2000 and March 20, 2002.

But the prize went unclaimed and the conjecture remains open.

It appears that the closest we can get is a result by J. R. Chen in the 70’s:

\[ 2n = p_1 + p_2p_3. \]