1. (5 marks) Explain the term *speculation*, and the roles it plays in mathematical thinking. Give an example to support your discussion.

Answer:
Answer:
2. (5 marks) Starting with the letter $b$ (step 1), consider the rewriting rules

$$b \rightarrow a; \quad a \rightarrow ab.$$  

For example, the first few ‘words’ formed by the above rewriting rules are

$$b, \ a, \ ab, \ aba, \ abaab, \ abaababa, \ ...$$

(a) In anyone of the words so obtained, can the letter $b$ be adjacent to another $b$? That is, can we find a word of the form $\cdots b b \cdots$? Justify your answers.

(b) Let $F(n)$ be the number of letters in the $n$-th word ($n$ is a positive integer). For instance,

$$F(1) = 1, \ F(2) = 1, \ F(3) = 2, \ F(4) = 3, \ F(5) = 5, \ F(6) = 8, \ ...$$

Prove that $F(n + 1) = F(n) + F(n - 1)$ for all positive integers $n \geq 2$.

Answer:
Answer:
3. (a) (5 marks) Find a formula (in terms of $n$) for the sum

$$1 + 3 + 5 + \cdots + (2n + 1).$$

[Hint: Apply the idea of ‘retrograding’ $(2n + 1) + \cdots + 5 + 3 + 1.$] Here $n$ is a positive integer. You are NOT required to prove the answers. Instead, provide a geometric interpretation of the result.

(b) (5 marks) Assume that you do NOT know the formula for the sum $1^2 + 2^2 + \cdots + n^2$ (that is, you are not allowed to use the formula). Formulate a conjecture on the form of a formula (in terms of the positive integer $n$) for the sum

$$1^2 + 3^2 + 5^2 + \cdots + (2n + 1)^2.$$

Prove your assertions. (Beware of the first term $1^2 = (2\cdot0+1)^2$. That is, it corresponds to $n = 0$.)

Answer:
Answer:

Please write your name on any additional sheets and staple them to the test paper.