Question 1.
(a) Number of choices for the hundreds, tens and units positions are 5, 5 and 4, respectively. Hence the number of 3-digit numbers formed = \(5 \times 5 \times 4 = 100\).
(b) Number of choices for the unit, hundreds and tens positions are 1, 4 and 4, respectively. Hence the number of odd 3-digit numbers formed = \(4 \times 4 \times 1 = 16\).
(c) Number of odd 3-digit numbers > 620 with hundreds position > 6: = \(1 \times 4 \times 1 = 4\).
Number of odd 3-digit numbers > 620 with hundreds position being 6: = \(1 \times 3 \times 1 = 3\).
Hence the number of 3-digit numbers > 620 = 4 + 3 = 7.

Question 2.
(a) \(8P_8 = 8! = 40320\).
(b) Let \(A, B, C, D\) represent the four couples. Number of ways to permute these four couples = \(4P_4 = 4!\).

For each of these permutations, we can permute the husband and wife in each couple, hence the number of ways to permute = \(2! \times 2! \times 2!\).

Therefore the number of ways that they can be seated if each couple is to sit together = \(4! \times (2! \times 2! \times 2!) = 384\).
(c) Number of ways to permute husbands = 4! and number of ways to permute wives = 4!.

Hence the number of ways that they can be seated together if all the men sit together to the right of all the women = 4! \times 4! = 576.

Question 3.
(a) \(n = 7\) and \(r = 5\). Number of choices is given by \(\left(\begin{array}{l}7 \\ 5 \end{array}\right) = \frac{7!}{5!2!} = 21\).
(b) Number of ways to choose three questions from the remaining 5 questions equals \(\left(\begin{array}{l}5 \\ 3 \end{array}\right) = \frac{5!}{3!2!} = 10\).

(c) Number of choices for selecting 1 question from the first 2 questions and 4 from the remaining 5 questions equals \(\left(\begin{array}{l}2 \\ 1 \end{array}\right) \times \left(\begin{array}{l}5 \\ 4 \end{array}\right) = 2 \times 5 = 10\).
Number of choices for selecting 2 question from the first 2 questions and 3 from the remaining 5 questions equals
\[
\binom{2}{2} \cdot \binom{5}{3} = 1 \cdot 10 = 10.
\]

Therefore the number of choices if at least one of the first two questions must be answered = 10 + 10 = 20.

(d) \( \binom{3}{2} \binom{4}{3} = 3 \cdot 4 = 12 \).

**Question 4.**

Note that \( \mathbb{P} \{ \text{exactly one of the events } E \text{ or } F \text{ occurs} \} = \mathbb{P}[E F^c \cup E^c F] \), of which \( E F^c \) and \( E^c F \) are mutually disjoint. Thus, RHS equals \( \mathbb{P}[E F^c] + \mathbb{P}[E^c F] \). Moreover,

\[
\mathbb{P}[E F^c] = \mathbb{P}[E] - \mathbb{P}[EF] \quad \text{and} \quad \mathbb{P}[E^c F] = \mathbb{P}[F] - \mathbb{P}[EF].
\]

We conclude that the desired probability is given by \( \mathbb{P}(E) + \mathbb{P}(F) - 2\mathbb{P}(EF) \).

**Question 5.**

(a) \( \binom{9}{1} \times \binom{27}{1} = 9(27) = 243 \).

(b) \( \binom{9}{1} \times \binom{27}{1} \times \binom{15}{1} = 9(27)(15) = 3645 \approx 10 \) (years).

**Question 6.**

(a) Number of possible permutations = 5! = 120.

Among all possible permutations, the number of permutations begin with a consonant = 3 \times 4! = 72.

Hence the probability that the permutation begins with a consonant = \( \frac{3(4!)}{5!} = \frac{3}{5} \).

(b) The probability that the permutation ends with a vowel = \( \frac{4!(2)}{5!} = \frac{2}{5} \).

(c) The probability that the permutation has the consonants and vowels alternating = \( \frac{3!2!}{5!} = \frac{1}{10} \).

**Question 7.**

The problem is equivalent to placing the right key in the \( r \)-th position and then permute the \( n-1 \) positions.

The total number of possible permutations is \( n! \). Hence the probability that the \( r \)-th key is the correct key is given by

\[
\frac{(n-1)!}{n!} = \frac{1}{n}.
\]
**Question 8.**

Let
\[ A = \{ \text{the factory to be set up in Shandong} \}, \quad \text{and} \]
\[ B = \{ \text{the factory to be set up in Jiangsu} \}. \]

Given that \( \mathbb{P}(A) = 0.7, \mathbb{P}(B) = 0.4, \) and \( \mathbb{P}(A \cup B) = 0.8, \) we get
(a) \( \mathbb{P}(A \cap B) = \mathbb{P}(A) + \mathbb{P}(B) - \mathbb{P}(A \cup B) = 0.7 + 0.4 - 0.8 = 0.3; \) and
(b) \( \mathbb{P}(A' \cap B') = 1 - \mathbb{P}(A \cup B) = 1 - 0.8 = 0.2. \)

**Question 9.**

(a) If \( A \) and \( B \) are mutually exclusive, then
\[ 0.8 = \mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B) = 0.4 + \mathbb{P}(B), \]
thus, \( \mathbb{P}(B) = 0.4. \)
(b) If \( A \) and \( B \) are independent, then \( \mathbb{P}(A \cap B) = \mathbb{P}(A) \mathbb{P}(B). \) Obviously,
\[ 0.8 = \mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B) - \mathbb{P}(A \cap B) = 0.4 + \mathbb{P}(B) + (0.4) \mathbb{P}(B), \]
which in turn implies that \( \mathbb{P}(B) = \frac{2}{3}. \) At last,
\[ \mathbb{P}(B \mid A) = \mathbb{P}(B) = 2/3. \]

**Question 10.**

(a) \( \mathbb{P}\{\text{all different}\} = \frac{(365)(364)(363) \cdots (356)}{(365)^{10}} = 0.8831. \)
\[ \mathbb{P}\{\text{at least two the same}\} = 1 - 0.8831 = 0.1169. \]

(b) Let \( A \) be the event that at least two have the same last three digits in their IC numbers. Then,
\[ \mathbb{P}(A) = 1 - \mathbb{P}\{\text{all different}\} \]
\[ = 1 - \frac{(1000)(999)(998) \cdots (991)}{(1000)^{10}} \]
\[ = 1 - 0.9559 = 0.0441. \]

Since the two “coincidence” are independent, therefore
\[ \mathbb{P}(\text{at least one “coincidence”}) = \mathbb{P}(\{BD coincidnee\} \cup \{IC coincidence\}) \]
\[ = 0.1169 + 0.0441 - (0.1169)(0.0441) \]
\[ = 0.1558. \]
**Question 11.**

Let $P$ denote the event the woman is pregnant and $T$ the test result is positive. By assumption,

$$P(P) = 0.75, \quad P(T \mid P^c) = 0.02, \quad \text{and} \quad P(T \mid P) = 0.99.$$ 

Hence, by the law of total probability,

$$P(T) = P(P) P(T \mid P) + P(P^c) P(T \mid P^c)$$

$$= 0.75 \times 0.99 + 0.25 \times 0.02 = 0.7475.$$ 

By Bayes’ rule,

$$P(P \mid T) = \frac{P(P \cap T)}{P(T)} = \frac{0.75 \times 0.99}{0.7475} = 0.9933.$$ 

**Question 12.**

Let $A_i, i = 1, 2, \ldots, 6$ denote the event that component $i$ works.

$$P(\{\text{system works}\}) = P(\{(A_1 \cup A_2) \cap ((A_3 \cap A_4) \cup (A_5 \cap A_6))\})$$

$$= P(\{A_1 \cup A_2\}) \cdot P(\{(A_3 \cap A_4) \cup (A_5 \cap A_6)\})\cdot$$

But

$$P(A_1 \cup A_2) = 0.9 + 0.9 - (0.9)^2 = 0.99,$$

and

$$P(\{(A_3 \cap A_4) \cup (A_5 \cap A_6)\}) = P(A_3 \cap A_4) + P(A_5 \cap A_6)$$

$$- P(\{(A_3 \cap A_4) \cap (A_5 \cap A_6)\}) = 0.9^2 + 0.9^2 - 0.9^4 = 0.9639.$$ 

Hence,

$$P(\{\text{system works}\}) = 0.99 \times 0.9639 = 0.9543.$$ 

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