1. A random sample of size 9 was drawn from a population. It was found that this random sample had a mean of 24 and a standard deviation of 4.1. It was claimed that the samples were from a normal population with mean 20. Is such a claim “reasonable”? Explain!

2. A taxi company is trying to decide whether to purchase brand A or brand B tires for its fleet of taxis. To estimate the difference in the two brands, an experiment is conducted using 12 of each brand. The tires are run until they wear out. The results are (in kilometers)

<table>
<thead>
<tr>
<th>Brand A</th>
<th>Brand B</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{x}_1 = 38,100$</td>
<td>$\bar{x}_2 = 36,300$</td>
</tr>
<tr>
<td>$s_1 = 6100$</td>
<td>$s_2 = 5000$</td>
</tr>
</tbody>
</table>

Would you agree with the claim that $\mu_1 - \mu_2 = 0$ kilometers, assuming the populations to be normally distributed with equal variances?

HINT: Make use of the statistic

$$T = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{S_p \sqrt{(1/n_1) + (1/n_2)}},$$

where

$$S_p^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}.$$

3. Choose at random a sample of size $n_1 = 2$ with replacement from the first set \{2, 3, 7\}. Denote by $\bar{X}_1$ the mean of the sample. Similarly, choose at random a sample of size $n_2 = 2$ with replacement from the second set \{1, 1, 3\}. Let $\bar{X}_2$ be the mean of the second sample.

(i) Find the sampling distribution of $\bar{X}_1 - \bar{X}_2$.

(ii) Find $\mu_{\bar{X}_1 - \bar{X}_2}$ and $\sigma^2_{\bar{X}_1 - \bar{X}_2}$, by using the sampling distribution of $\bar{X}_1 - \bar{X}_2$ derived in (i).

(iii) Verify now the formulas

$$\mu_{\bar{X}_1 - \bar{X}_2} = \mu_1 - \mu_2 \text{ and } \sigma^2_{\bar{X}_1 - \bar{X}_2} = \frac{\sigma^2_1}{n_1} + \frac{\sigma^2_2}{n_2},$$

where $E(X_i) = \mu_i$ and $\text{Var}(X_i) = \sigma_i$ for $i = 1, 2$. 

1
4. An electrical firm manufactures light bulbs that have a length of life that is approximately normally distributed with a standard deviation of 40 hours. If a sample of 30 bulbs has an average life of 780 hours, find a 96% confidence interval for the population mean of all bulbs produced by this firm.

5. How large a sample is needed in Question 4 above if we want to be 96% confident that our sample mean will be within 5 hours of the true mean?

6. Let $X$ be a random variable with mean $\mu$ and variance $\sigma^2$. Let $(X_1, X_2, \cdots, X_n)$ be a random sample from $X$ of size $n \geq 2$. There are many other estimators of the unknown parameter $\sigma^2$ in addition to $S^2 = \frac{1}{n-1} \sum_{i=1}^{n} (X_i - \overline{X})^2$.

Show that the estimator $C \sum_{i=1}^{n-1} (X_{i+1} - X_i)^2$ is also an unbiased estimator of $\sigma^2$ for an appropriate choice of the constant $C$. Find the value of $C$.

7. In measuring reaction time, a psychologist estimates that the standard deviation is 0.05 second. How large a sample of measurements must be taken in order to be
   (a) 95%, and
   (b) 99%
   confident that the error of his estimate will not exceed 0.01 second?
   (c) Suppose that 10 independent 95% confidence intervals are constructed. What is the probability that only one confidence interval does not include the population mean?

8. The mean and standard deviation for the quality point averages of a randomly sample of 36 college seniors are calculated to be 2.6 and 0.3, respectively.
   (a) Find the 95% and 99% confidence intervals for the mean of the entire senior class.
   (b) Suppose that 2.6 is used as an estimate of the average.
      With what probability can it be said that this estimate is “off” by at most 0.02?
      What can be said with 98% confidence about the maximum size of the error?
   (c) How large a sample is required if we want to be 95% confident that our estimate of $\mu$ is off by less than 0.05?

9. The contents of seven similar containers of sulfuric acid are 9.8, 10.2, 10.4, 9.8, 10.0, 10.2, and 9.6 liters. Assume that the population is normally distributed.
   (a) Find a 95% confidence interval for the mean of all such containers.
(b) Find a 95% confidence interval for the mean of all such containers if it is given that the population standard deviation is 0.2828.

10. Five people selected at random had their breathing capacity measured before and after a certain treatment as follows:

*Breathing Capacity*

<table>
<thead>
<tr>
<th>Person</th>
<th>Before (X)</th>
<th>After (Y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>2750</td>
<td>2850</td>
</tr>
<tr>
<td>B</td>
<td>2360</td>
<td>2380</td>
</tr>
<tr>
<td>C</td>
<td>2950</td>
<td>2930</td>
</tr>
<tr>
<td>D</td>
<td>2830</td>
<td>2860</td>
</tr>
<tr>
<td>E</td>
<td>2250</td>
<td>2320</td>
</tr>
</tbody>
</table>

Let \( \mu_X \) (and \( \mu_Y \)) be the mean capacity of the whole population before (and after) treatment. Construct a 95% confidence interval for \( \mu_X - \mu_Y \).

11. In a random sample of 10 football players, the average age was 27 and the standard deviation was 5.5. In a random sample of 20 hockey players, the average was 25 and the standard deviation was 4.5. Find a 90% confidence interval for the difference in the population means, assuming the two populations are approximately normal with equal variance.

12. A manufacturer of car batteries claims that his batteries will last, on average, 3 years with a variance of 1 year. If 5 of these batteries have lifetimes of 1.9, 2.4, 3.0, 3.5, and 4.2 years, construct a 95% confidence interval for \( \sigma^2 \) and decide if the manufacturer’s claim that \( \sigma^2 = 1 \) is valid. Assume that the population of battery lives to be approximately normally distributed.

13. (Refer to Ex. 2.) A taxi company is trying to decide whether to purchase brand A or brand B tires for its fleet of taxis. To estimate the difference in the two brands, an experiment is conducted using 12 of each brand. The tires are run until they wear out. The results are (in kilometers)

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(a) Compute a 95% confidence interval for \( \mu_1 - \mu_2 \), assuming the populations to be approximately normally distributed with equal variances.

(b) Construct a 90% confidence interval for \( \sigma_1^2/\sigma_2^2 \). Were we justified in assuming that \( \sigma_1^2 = \sigma_2^2 \) when we constructed a confidence interval for \( \mu_1 - \mu_2 \) in (a)?
14. Consider the following measurements of the heat-producing capacity of the coal produced by two mines (in millions of calories per ton).

<table>
<thead>
<tr>
<th>Mine 1</th>
<th>8260</th>
<th>8130</th>
<th>8350</th>
<th>8070</th>
<th>8340</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mine 2</td>
<td>7950</td>
<td>7890</td>
<td>7900</td>
<td>8140</td>
<td>7920</td>
</tr>
</tbody>
</table>

Can it be concluded that the two population variances are equal?

15. Given two random samples of size $n_1 = 9$ and $n_2 = 16$ from two independent normal populations with $\mu_1 = 64$, $\mu_2 = 59$, $s_1 = 6$, and $s_2 = 5$. Find a 98% confidence interval for $\sigma^2_1/\sigma^2_2$.

16. The following results were obtained for the length of life in the hours of 10 light bulbs: 1293, 1380, 1614, 1497, 1340, 1643, 1466, 1094, 1270, 1028. Find

(a) a 95% confidence interval for $\sigma^2$, and

(b) a 95% confidence interval for $\sigma$.

Assume that the life lengths are normally distributed.

**Answers to Selected Problems:**

1. It is **unlikely** to obtain such a random sample from a normal population. Suppose they were drawn from $N(20, \sigma^2)$. Then $T = \frac{\bar{X} - \mu}{S/\sqrt{n}}$ has a $t$-distribution with $n - 1$ degrees of freedom. Here $\mu = 20$, $n = 9$, while $\sigma$ is unknown, and $\bar{x} = 24$ and $s = 4.1$. By looking up the Table 7 (Percentage Points of the $t$-Distribution), one sees that 95% of the $t$-values with 8 degrees of freedom fall between $-2.306$ and $2.306$.

2. $s_p = 5577.1857$, $t \approx 0.7906$.
3. (b) $\frac{7}{3}$; $\frac{25}{9}$.

4. $(765, 795)$.
5. $n = 271$.
6. $C = \frac{1}{2(n-1)}$.

7. (a) 97; (b) 166; (c) 0.3151.

8. (a) $(2.502, 2.698)$; $(2.471, 2.729)$; (b) (i) 0.3108; (ii) 0.1163; (c) 139.

9. (a) $(9.74, 10.26)$; (b) $(9.79, 10.21)$.

10. $(-17.56, 97.56)$. 11. $(-1.191, 5.191)$.

12. $0.293 < \sigma^2 < 6.736$; valid claim.
13(b). $0.238 < \sigma^2_1/\sigma^2_2 < 1.895$; yes.

14. Yes.
15. $(0.359, 8.024)$.

16(a). $(19393.18, 136635.74)$. 