1. Let \( p \) denote the proportion of all registered voters in a certain city who favor candidate A over candidate B in the race for mayor. Consider testing \( H_0 : p = 0.5 \) versus \( H_1 : p \neq 0.5 \) based on a random sample of 50 registered voters. Let \( X \) denote the number in the sample who favor A and \( x \) represent the observed value of \( X \).

(a) Which of the following rejection regions is most appropriate and why?

\[
R_1 = \{ x : x \leq 18 \text{ or } x \geq 32 \}, \quad R_2 = \{ x : x \leq 20 \}, \quad R_3 = \{ x : x \geq 30 \}.
\]

(b) What is the probability distribution of the test statistics \( X \) when \( H_0 \) is true? Use it to compute the probability of a type I error for the selected region.

(c) Compute the probability of a type II error for the selected region when \( p = 0.4 \).

(d) Using the selected region, what would you conclude if 16 of the 50 queried favored A?

2. The sample standard deviation of sodium concentration in the whole blood (mEq/L) for \( n_1 = 25 \) marine eels was found to be \( s_1 = 40.5 \), while the sample standard deviation of concentration for \( n_2 = 19 \) freshwater eels was \( s_2 = 32.1 \). Assuming normality of the two concentration distributions, test at the 5% significance level to see if the data suggests any difference between concentration variances for the two types of eels.

3. The melting point of each of 16 samples of a certain brand of hydrogenated vegetable oil was determined, resulting in \( \bar{x} = 94.32 \). Assume that the distribution of melting point is normal with \( \sigma = 1.20 \).

(a) Test \( H_0 : \mu = 95 \) against \( H_1 : \mu \neq 95 \) using \( \alpha = 0.01 \).

(b) If a 1% significance level is used in the test, what is \( \beta(94) \), the probability of type II error when \( \mu = 94 \)?

(c) What value of \( n \) is necessary to ensure that \( \beta(94) = 0.1 \) when \( \alpha = 0.01 \)?

4. In an experiment designed to measure the time necessary for an inspector’s eyes to become used to the reduced amount of light necessary for penetrant inspection, the sample average time for \( n = 9 \) inspectors was 6.32 seconds and the sample standard deviation was 1.65 second. It has previously been assumed that the average adaption time was at least 7 seconds. Assuming adapting time to be normally distributed, do the data contradict prior belief? Use \( \alpha = 0.05 \).

5. A study reports results of an experiment to compare handling ability for two cars having quite different lengths, wheelbases, and the turning radii. The observations
are time in seconds required for drivers to parallel park each car.

<table>
<thead>
<tr>
<th>Driver</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Car A</td>
<td>37.0</td>
<td>25.8</td>
<td>16.2</td>
<td>24.2</td>
<td>22.0</td>
<td>33.4</td>
<td>23.8</td>
<td>58.2</td>
<td>33.6</td>
<td>24.4</td>
<td>23.4</td>
<td>21.2</td>
</tr>
<tr>
<td>Car B</td>
<td>17.8</td>
<td>20.2</td>
<td>16.8</td>
<td>41.4</td>
<td>21.4</td>
<td>38.4</td>
<td>16.8</td>
<td>32.2</td>
<td>27.8</td>
<td>23.2</td>
<td>29.6</td>
<td>20.6</td>
</tr>
</tbody>
</table>

Do the data suggest that the average person will more easily handle one car than the other? Test the relevant hypotheses using \( \alpha = 0.05 \).

6. An experiment to compare the tension bond strength of polymer latex modified mortar to that of unmodified mortar resulted in \( \bar{\tau}_1 = 18.12 \text{ kgf/cm}^2 \) and \( s_1 = 1.6 \) for the modified mortar \( (n_1 = 40) \) and \( \bar{\tau}_2 = 16.87 \text{ kgf/cm}^2 \) and \( s_2 = 1.4 \) for the unmodified mortar \( (n_2 = 32) \). Let \( \mu_1 \) and \( \mu_2 \) be the true average tension bond strengths for the modified and unmodified mortars, respectively. Test \( H_0 : \mu_1 - \mu_2 = 0 \) versus \( H_1 : \mu_1 - \mu_2 > 0 \) at \( \alpha = 0.01 \).

7. The following data represent the running times (in minutes) of films produced by 2 motion-picture companies:

<table>
<thead>
<tr>
<th>Company 1</th>
<th>102</th>
<th>86</th>
<th>98</th>
<th>109</th>
<th>92</th>
</tr>
</thead>
<tbody>
<tr>
<td>Company 2</td>
<td>112</td>
<td>145</td>
<td>97</td>
<td>134</td>
<td>92</td>
</tr>
</tbody>
</table>

Test the hypothesis that the average running time of films produced by Company 2 exceeds the average running time of films produced by Company 1 by 10 minutes against the one-sided alternative that the difference is more than 10 minutes. Use a 5% level of significance and assume the distributions of times to be approximately normal with equal variances.

8. Past data indicate that the amount of money contributed by the working residents of a large city to a volunteer rescue squad is normal random variable with a standard deviation of \$1.40. It has been suggested that the contributions to the rescue squad from just the employees of the sanitation department are much more variable. If the contributions of a random sample of 12 employees from the sanitation department had a standard deviation of \$1.75, can we conclude at the 1\% significance level that the standard deviation of the contributions of all sanitation workers is greater than that of all workers living in that city?
Answers to Selected Problems:

1. (a) $R_1$. (b) $B(50, 0.5)$, 0.065. (c) 0.7629. (d) Reject $H_0$.
2. $F = 0.4158$, do not reject $H_0 : \sigma_1^2 = \sigma_2^2$.
3. (a) $z = -2.2667$, do not reject $H_0 : \mu = 95$. (b) 0.2244. (c) 22.
4. $t = -1.2364$, do not reject $H_0 : \mu = 7$.
5. $t = 0.9446$, do not reject $H_0 : \mu_0 = \mu_1 - \mu_2 = 0$.
6. $z = 3.532$, reject $H_0 : \mu_1 - \mu_2 = 0$.
7. $t = 0.402$, do not reject $H_0 : \mu_2 - \mu_1 = 10$.
8. $\chi^2 = 17.1875$, do not reject $H_0 : \sigma = 1.4$. 