1. Determine whether the following sequences of functions converge uniformly on the indicated intervals. Justify your answers.

(a) \( F_n(x) = \frac{n^2}{n^2 + x^2}, \ x \in [0, 1]. \)

(b) \( F_n(x) = x^n(1-x), \ x \in [0, 1]. \)

(c) \( f_n(x) = \frac{n \ln x}{x^n}, \ x \in [1, \infty). \)

(d) \( f_n(x) = \frac{n \ln x \cos nx}{x^n}, \ x \in [4, \infty). \)

(e) \( F_n(x) = \frac{n^2}{n^2 + x^2}, \ x \in [0, +\infty). \)

2. Prove that each of the following series of functions converges uniformly on the indicated interval.

i) \( \sum_{n=1}^{\infty} \frac{\cos nx}{n^2 + x^2}, \ x \in (-\infty, +\infty). \)

ii) \( \sum_{n=1}^{\infty} \frac{1}{1 + n^3x^2}, \ x \in [2, \infty). \)

iii) \( \sum_{n=1}^{\infty} \frac{xe^{-nx}}{n^2}, \ x \in (0, \infty). \)

3. Let \( \sum_{n=1}^{\infty} f_n(x) \) and \( \sum_{n=1}^{\infty} g_n(x) \) be series of functions on an interval \( I \) with

\[ |f_n(x)| \leq g_n(x) \]

for all \( x \in I \) and \( n \geq 1 \). Suppose that the series of functions \( \sum_{n=1}^{\infty} g_n(x) \) converges uniformly. Show that the series of functions \( \sum_{n=1}^{\infty} f_n(x) \) converges uniformly.

4. Does the series of functions \( \sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n^2} \) converge uniformly on the interval \((0, +\infty)\)? Justify your answer.