Question 1. [4 points, 1 for each part] For each of the following, determine all values of $x$ for which the given series converges.

(a) \[ \sum_{k=1}^{\infty} \frac{(1 - 2x)^k}{k}. \]

(b) \[ \sum_{k=1}^{\infty} \frac{3^k}{k^3} (2x + 1)^k. \]

(c) \[ \sum_{k=1}^{\infty} \frac{3^k x^k}{2^k (1 - x)^k}, \quad x \neq 1. \]

(Hint: Let \( t = \frac{3x}{2(1 - x)} \).)

(d) \[ \sum_{k=1}^{\infty} \frac{1}{k^2}, \quad x \neq 0. \]

(Hint: Let \( t = \frac{1}{x} \).)

Question 2. [4 points, 1 for each part] Using any applicable method, find the Taylor series of each of the following functions at the indicated point, and specify the interval on which the series converges to the function.

(a) \( f(x) = \cos x^2, \quad x_0 = 0. \)

(b) \( f(x) = \ln \left( \frac{1 + x}{1 - x} \right), \quad x_0 = 0. \)

(c) \( f(x) = \sqrt{x}, \quad x_0 = 1. \)

(d) \( f(x) = \frac{x^2}{1 - x^2}, \quad x_0 = 0. \)

Question 3. [2 points, 1 for each part] Use series to estimate the integral’s value

\[ \int_0^{0.1} \arctan x^2 \, dx \]

with an error of magnitude less than \( 10^{-8} \).

(b) Let \( f(x) = x^3 \sin x^9. \) Find \( f^{(30)}(0) \).