1. Denote the set of rational numbers by \( \mathbb{Q} \). Consider the set 
\[ S = \{ x \in \mathbb{Q} \mid 0 \leq x < 1 \}. \]
Find \( \sup S \) and \( \inf S \). Justify your answers.

2. Let \( A \) and \( B \) be two non-empty bounded set of real numbers such that \( A \subseteq B \). Show that \( \inf A \geq \inf B \).

3. Let \( A \) and \( B \) be two non-empty bounded set of real numbers
   i) Show that \( \sup A \cup B = \max\{\sup A, \sup B\} \).
   ii) Is it true that \( \sup A \cap B = \min\{\sup A, \sup B\} \)? Justify your answer.

4. Consider the sequence \( \{a_n\} \) defined recursively by 
\[ a_1 = 2, \quad a_n = \sqrt{6 + a_{n-1}}, \quad n = 2, 3, 4, \ldots \]
   i) Show that \( 2 \leq a_n \leq 3 \) for all \( n \).
   ii) Show that \( \{a_n\} \) is monotone increasing.
   iii) Using parts i) and ii), show that \( \{a_n\} \) converges, and find its limit.

5. Consider the sequence \( \{x_n\} \) defined recursively by
\[ x_1 = \frac{3}{4}, \quad x_{n+1} = 2x_n - x_n^2, \quad n = 1, 2, 3, \ldots \]
Show that \( \{x_n\} \) converges, and find its limit. (Hint: Show that \( 0 \leq x_n \leq 1 \) for all \( n \) and \( \{x_n\} \) is monotone increasing.)

6. Find the \( \lim_{n \to \infty} \) and \( \lim_{n \to \infty} \) of the following sequences.
   (a) \( \left\{ 4 + \cos \frac{n\pi}{2} \right\} \)
   (b) \( \left\{ \frac{1 + (-1)^n}{n} \right\} \)