1. Show that the series $\sum_{n=1}^{\infty} (-1)^n \frac{\cos n}{2n}$ is absolutely convergent.

2. For each of the following series, determine whether the series is absolutely convergent, conditionally convergent or divergent. Justify your answers.
   (a) $\sum_{n=1}^{\infty} (-1)^n \frac{3}{2n+1}$
   (b) $\sum_{n=1}^{\infty} (-1)^n \frac{n}{4n+3}$
   (c) $\sum_{n=1}^{\infty} (-1)^n \frac{(1 + 2n)}{(3 + 4n)}$
   (d) $\sum_{n=2}^{\infty} (-1)^{n+1} \frac{\cos n}{n(ln n)^2}$

3. Estimate the infinite sum $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n^3}$ such that the error is within 0.001.

4. For each of the following sequence of functions, determine whether converges pointwise to a function, and find the limiting function if it exists. Justify your answers.
   (a) $\left\{\left(1 + \frac{x}{n}\right)^{nx}\right\}$, $x \in (-\infty, +\infty)$.
   (b) $\{x^{n+1}\}$, $x \in [-1, 1]$.
   (c) $\left\{x^{2n}\right\}$, $x \in [0, 1]$.

5. Let $\{F_n\}$ be a sequence of functions on an interval $I$. It is given that $\{F_n\}$ converges uniformly on some function $F$ on $I$. Suppose also that for each $n \in \mathbb{Z}^+$, there exists a real number $M_n > 0$ such that
   $$|F_n(x)| \leq M_n$$
   for all $x \in I$.

   Show that there exists a real number $M$ such that $|F(x)| \leq M$ for all $x \in I$. 