1. By computing derivatives, find the Taylor series of
   i) \( f(x) = e^{2x} \) at \( x = 3 \).
   ii) \( f(x) = \cos x \) at \( x = \frac{\pi}{3} \).
2. Find the Taylor series of \( \ln(1 + 2x^2) \) at \( x_0 = 0 \).
3. Using the Taylor Formula, show that \( \cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} \).
4. Use series to estimate the integral’s value
   \[ \int_0^{0.2} \sin x^2 dx \]
   with an error of magnitude less than \( 10^{-8} \).
5. Use series to evaluate the limits
   i) \( \lim_{y \to 0} \frac{\arctan y - \sin y}{y^3 \cos y} \).
   ii) \( \lim_{x \to \infty} x^2(e^{-1/x^2} - 1) \).

Review question of chapter 3.

6. Let \( \sum_{n=1}^{\infty} f_n(x) \) be a series of functions on an interval \( I \) and let \( \{g_n(x)\} \) be a sequence of functions on \( I \). Suppose that
   1) \( \sum_{n=1}^{\infty} |f_n(x)| \) converges uniformly on \( I \) and
   2) there exists a positive number \( M \) such that \( |g_n(x)| \leq M \) for all \( x \in I \) and all \( n \geq 1 \).
   Show that the series of functions \( \sum_{n=1}^{\infty} f_n(x)g_n(x) \) converges uniformly on \( I \).