1. ‘Classify’ the following differential equations, i.e., determine whether the following equations are homogeneous/non-homogeneous, linear/non-linear, 1st order/2nd order/⋯, ordinary differential equations:

a) $t \frac{dy}{dt} + y \sin t = 0;
\quad$

b) $(1 + t^2) \frac{dy}{dt} + 2ty + 3 = 0;
\quad$

c) $\frac{dy}{dt} + \frac{d^3y}{dt^3} = 3t;
\quad$

d) $4\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 6y = e^t \sin 2t;
\quad$

e) $u\frac{d^4u}{dt^4} + \left(\frac{du}{dt}\right)^2 = 0.$

2. Find the general solution of the following differential equations:

a) $\frac{dy}{dt} + y \sin t = 0;
\quad$

b) $(1 + t^2) \frac{dy}{dt} + ty = (1 + t^2)^{3/2}$

3. Solve the following initial/boundary value problems, and express your answers in the form “$y=y(t)$”.

a) $\frac{dy}{dt} + 4ty = 2te^{-2t^2}, \quad y(1) = 5.
\quad$

b)* $(1 + t^2) \frac{dy}{dt} - 2ty^3 = 0, \quad y(0) = \frac{1}{2}.
\quad$

c)* $\sqrt{1 + t^3} \frac{dy}{dt} - 6t^2e^{-y} = 0, \quad y(0) = 0.$

4. Find the general solution of the following differential equations

a) $\frac{dy}{dt} = \frac{y}{t} + \frac{y^2}{t^2}.
\quad$

b)* $\frac{dy}{dt} = \frac{y}{t} - \tan \frac{y}{t}.$

5. Find the the general solution of the following differential equations:

a) $\frac{d^2y}{dt^2} - 7\frac{dy}{dt} + 10y = 0;
\quad$

b) $\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 9y = 0;
\quad$

c) $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 3y = 0.$

This is the last tutorial. The questions with * will be gone through during lectures.
6. Solve the following initial value problem:
\[ \frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y = 0; \quad y(0) = 1, \quad y'(0) = 0. \]

7. Use the method of undetermined coefficients to find a particular solution of the following differential equations:
   a) \( y'' + 3y = 6t^3 - 6 \).
   b)* \( y'' + 3y = \cos(\sqrt{3}t) \).

8*. Find the general solution of the following differential equations:
   a) \( y'' - 3y' + 2y = 3e^{2t} + e^{3t} \).
   b) \( y'' + 4y = \cos 2t + \sin t \).
   c) \( y'' - 2y' + y = \frac{e^t}{t} \).

9*. Solve the following initial value problem:
\[ y'' + 4y' + 3y = 2e^{-3t}; \quad y(0) = 0, \quad y'(0) = 3. \]

10*. A tank containing \( S_0 \) lb of salt dissolved in 200 gallons of water. Starting at time \( t = 0 \), water containing \( \frac{1}{2} \) lb of salt per gallon enters the tank at rate of 4 gal/min, and well stirred solution leaves the tank at the same rate. Find the concentration of salt in the tank at any time \( t > 0 \).

Some suggested answers:
1 a) homogeneous linear 1st order ODE
1 b) non-homogeneous linear 1st ODE
1 c) non-homogeneous non-linear 3rd order ODE
1 d) non-homogeneous linear 2nd order ODE
1 e) homogeneous non-linear 4-th order ODE.
2 a) \( y = Ce^{\cos t} \)
2 b) \( y = \frac{1}{\sqrt{1 + t^2}} \left( t^3 + C \right) \)
3 a) \( y = e^{-2t^3} (t^2 + 5e^2 - 1) \)
3 b) \( y = \frac{1}{\sqrt{8 - 3 \ln(1 + t^2)}} \)
3 c) \( y = \ln(4\sqrt{1 + t^3} - 3) \)
4 a) \( y = -\ln(|t|) + C \)
4 b) \( y = t \arcsin \left( \frac{C}{t} \right) \)
5 a) \( y(t) = Ae^{2t} + Be^{3t} \)
5 b) \( y(t) = (At + B)e^{-3t} \)
5 c) \( y(t) = Ae^{-t} \cos \sqrt{2}t + Be^{-t} \sin \sqrt{2}t \)
6. \( y = e^{-t} + te^{-t} \)
7 a) \( y_p(t) = 2t^3 - 4t - 2 \)
7 b) \( y_p = \frac{1}{2\sqrt{3}} t \sin(\sqrt{3}t) \)
8 a) \( y(t) = Ae^t + Be^{2t} + 3te^{2t} + \frac{1}{2}e^{3t} \)
8 b) \( y(t) = A \cos 2t + B \sin 2t + \frac{t}{4} \sin 2t + \frac{1}{3} \sin t \)
8 c) \( y(t) = Ae^t + Bte^t + te^t \ln |t| \)
9. \( y(t) = 2e^{-t} - 2e^{-3t} - te^{-3t} \)
10. the concentration \( y(t) \) of salt in the tank is given by
\[
y(t) = \frac{S_0}{200} e^{-0.02t} + \frac{1}{2} (1 - e^{-0.02t}).
\]